

– Ultrasound motion imaging –

Color Doppler





by
Damien Garcia
INSERM researcher, CREATIS, Lyon, France

www.biomecardio.com
garcia.damien@gmail.com

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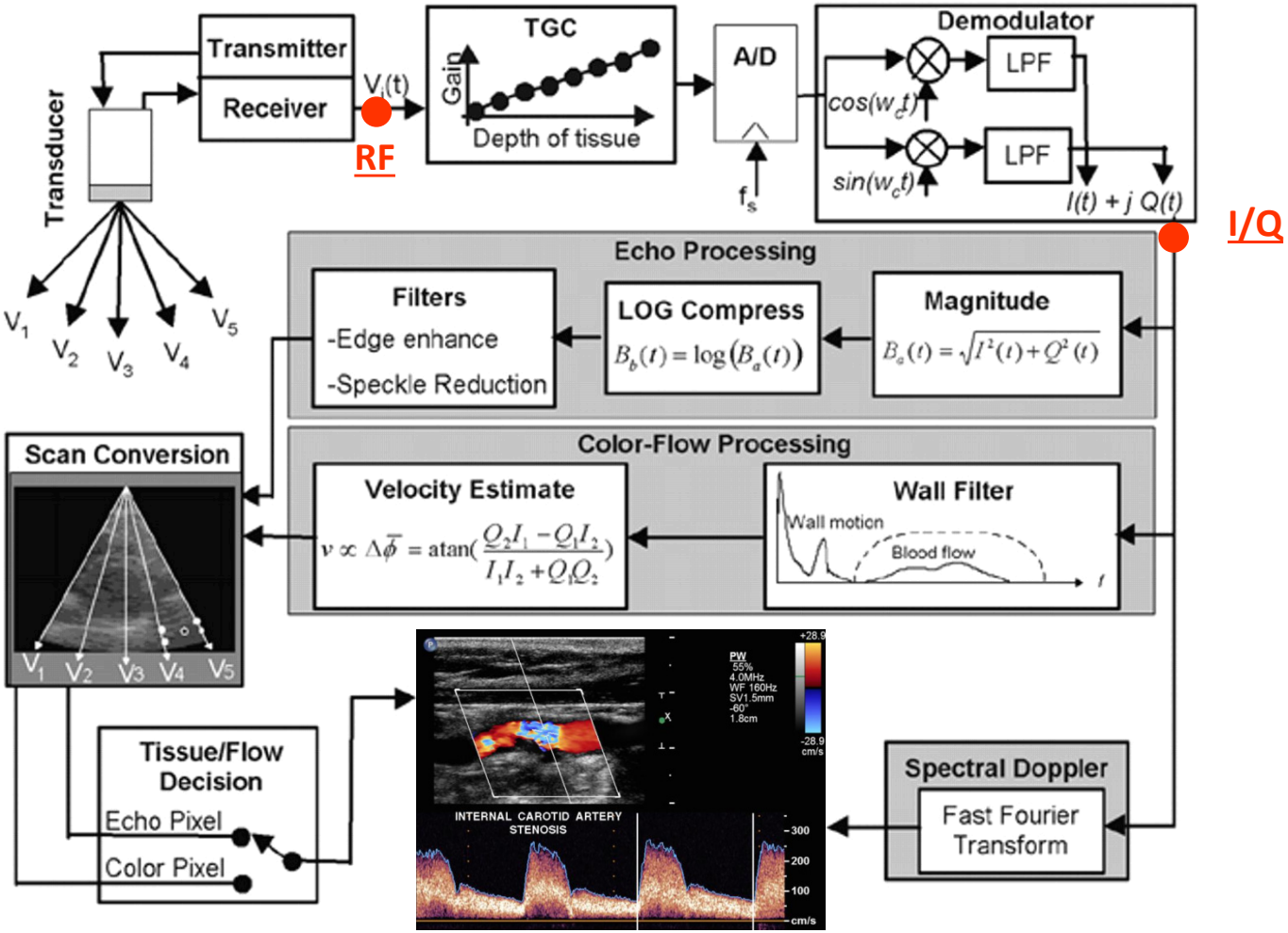
Disclaimer: The views expressed in this course are those of the author and do not necessarily reflect the multiple positions of the ultrasound community. The examples may contain errors and can carry an implied judgement due to author's preference for one side of an issue over another.
Be critical and take a step back while reading this document!

TOC

- RF & I/Q
- color Doppler
- vector Doppler
- clutter filtering & smoothing

RF & I/Q

an ultrasound scanner

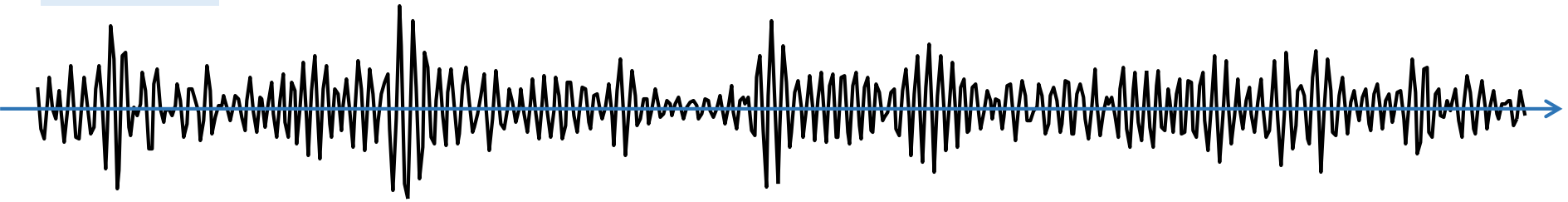


source: York et al.
Annu Rev Biomed Eng, 1999

RF = radiofrequency signal
I/Q = in-phase/quadrature

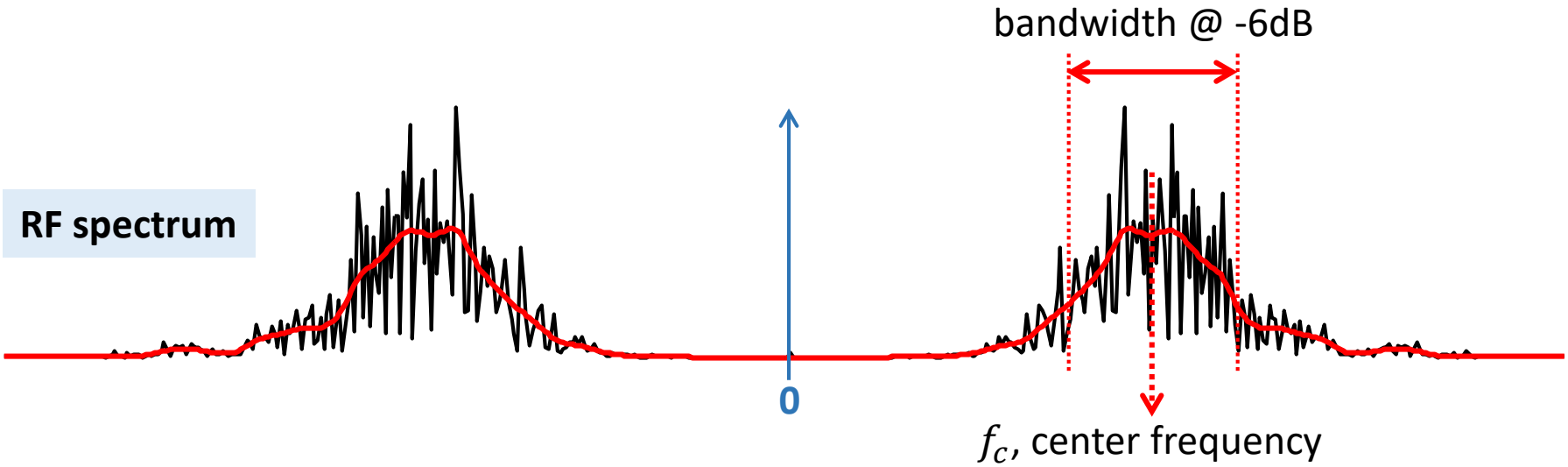
RF signal & its spectrum

RF signal

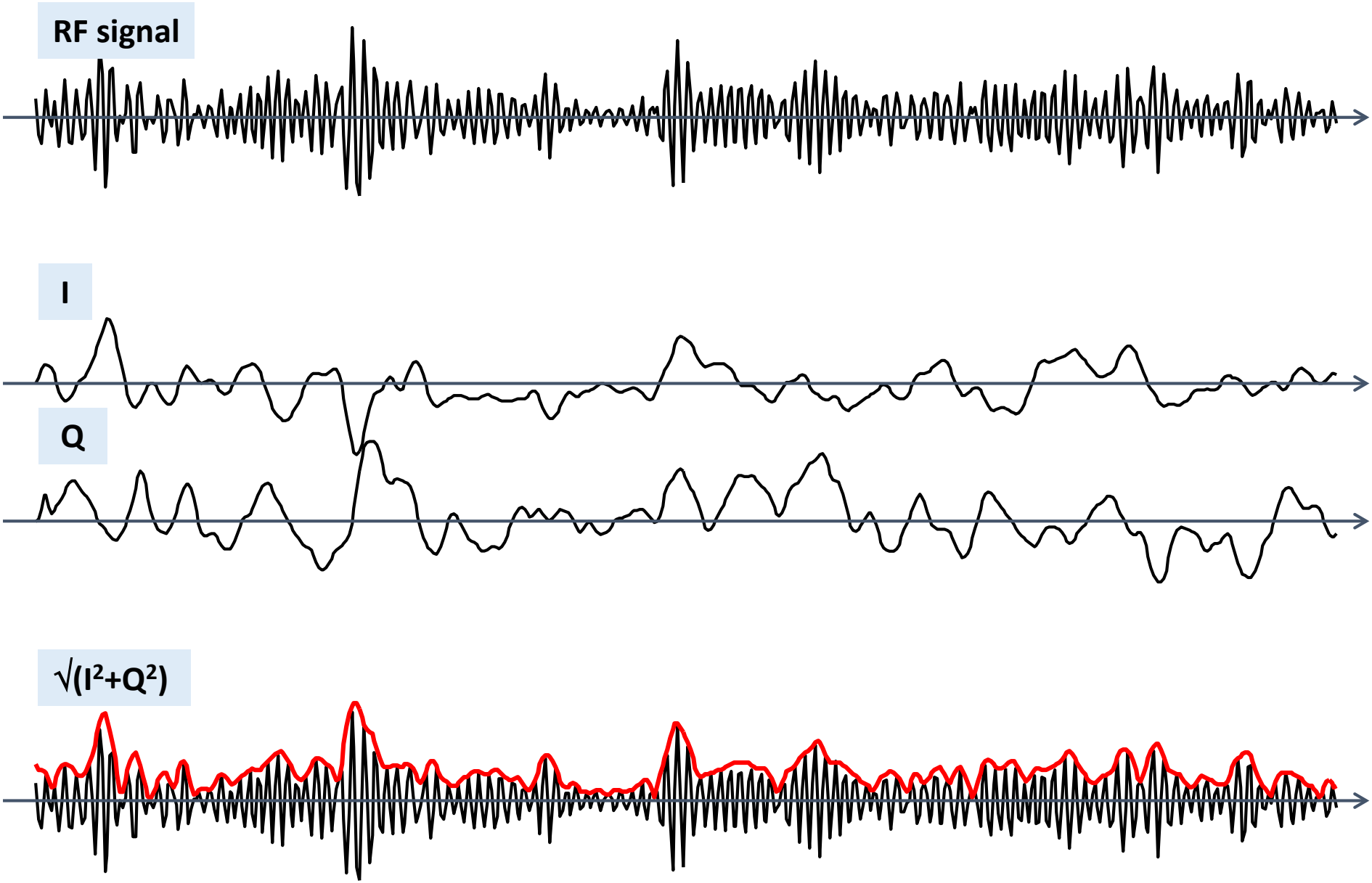


- the ultrasound RF signal is a **band-pass signal**

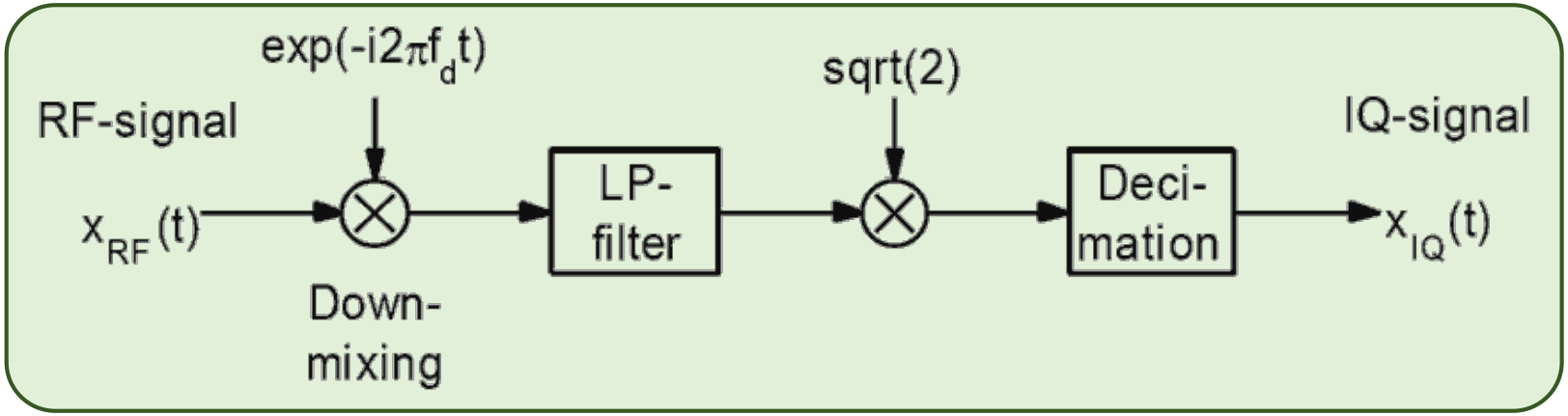
RF spectrum

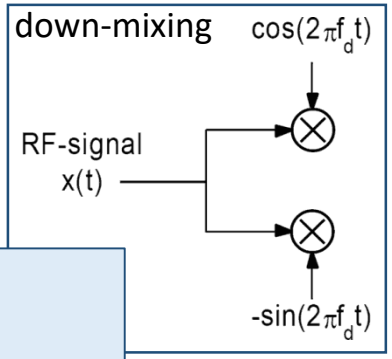


RF *vs.* I/Q signal



I/Q demodulation



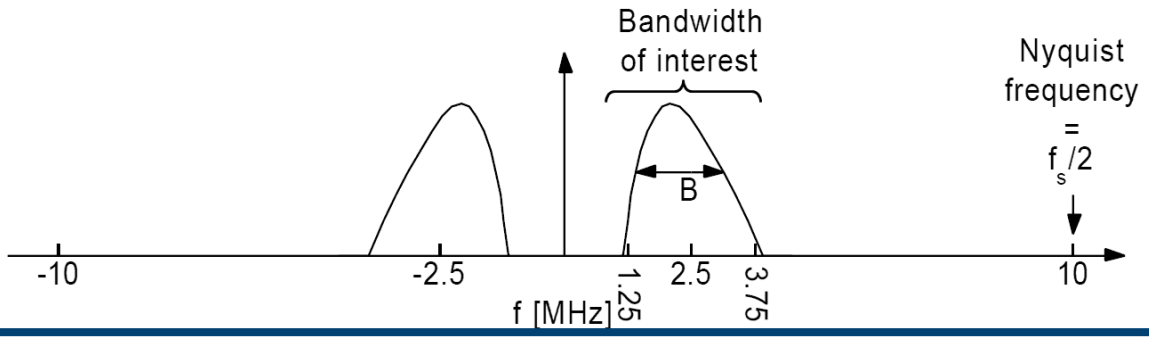


- f_d = demodulation frequency (center frequency)
- LP-filter = low-pass filter
- conservation of signal energy $\Rightarrow$ multiplication by $\sqrt{2}$
- decimation = downsampling

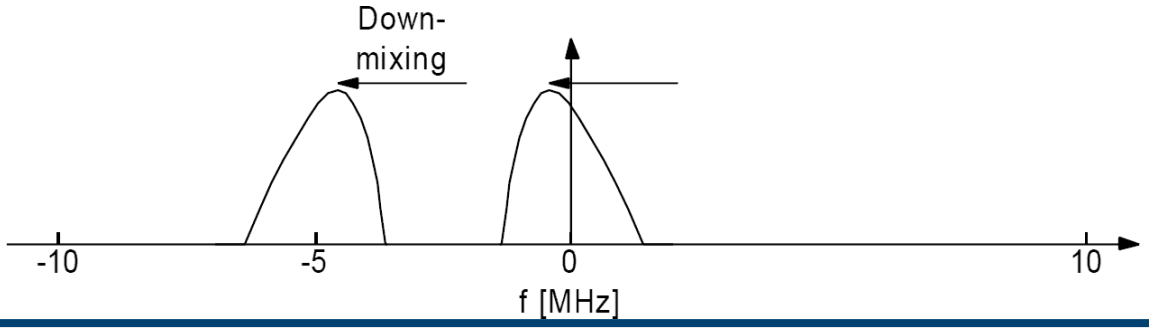
source: Kirkhorn, *Introduction to IQ-demodulation of RF data*. White paper, 1999

I/Q demodulation

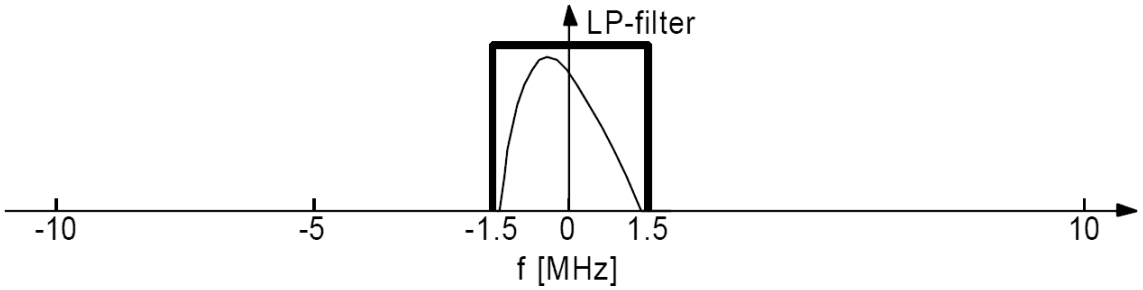
1. RF spectrum



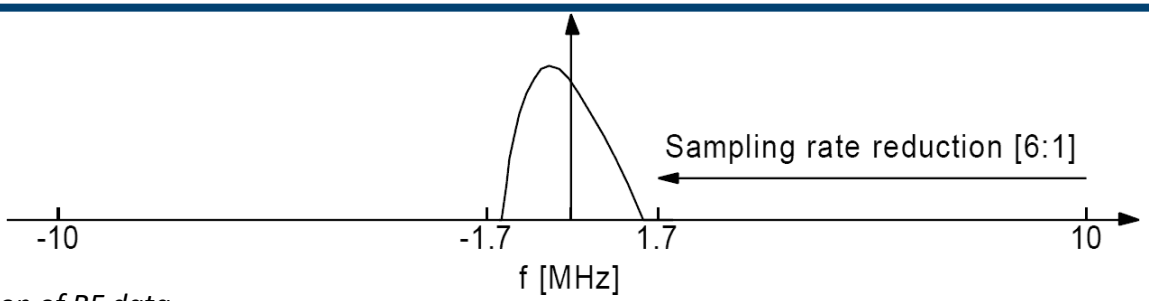
2. Down-mixing



3. Low-pass filtering



4. Decimation



source: Kirkhorn, *Introduction to IQ-demodulation of RF data*.
White paper, 1999

I/Q demodulation

- **I/Q demodulation of RF signals – a 5-line Matlab code**



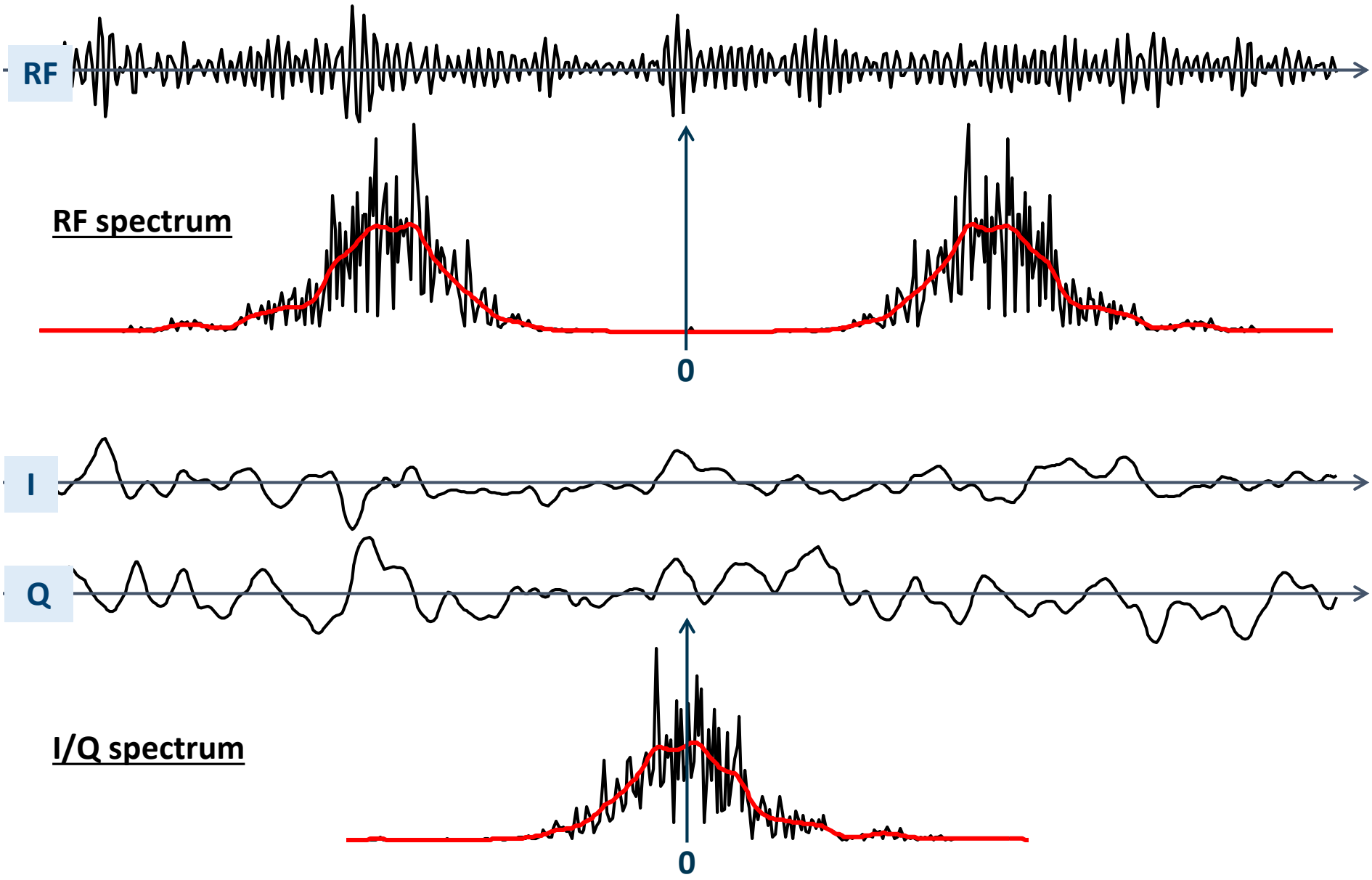
```
%-- RF = RF ultrasound signals
% RF = array whose columns contain the RF signals
% Fs = sampling frequency
% Fc = carrier (center) frequency

%-- Time column vector
N = size(RF,1); % number of time samples
t = (0:N-1)'/Fs;

%-- Down-mixing of the RF signals
IQ = RF.*exp(-1i*2*pi*Fc*t); % w/ Matlab version >9.1

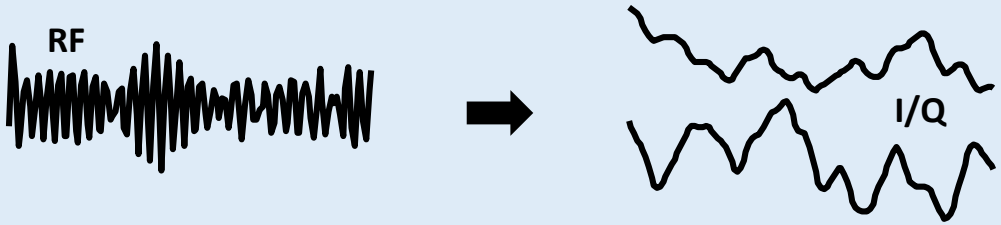
%-- Low-pass filter
% IQ = I/Q signals
[b,a] = butter(5,2*Fc/Fs);
IQ = filtfilt(b,a,IQ)*2; % *2 to preserve amplitudes
```

I/Q demodulation: recap

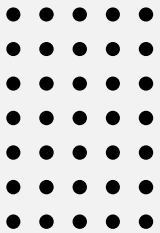


4 steps to generate an ultrasound image

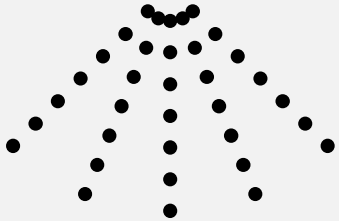
1. Demodulate the received RF signals to obtain I/Q signals



2. Choose a grid for your image



or



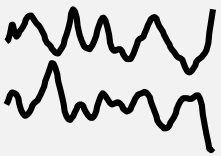
or what you need

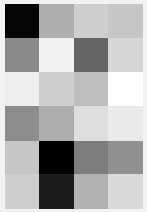
3. Beamform the I/Q signals (e.g. using a delay-and-sum)

```
IQ_beamformed = Mdas*IQ(:);
% Mdas: matrix of size [#grid_points #IQ_samples]
```

4. Take the amplitude then compress to obtain a B-mode image

beamformed **I/Q**



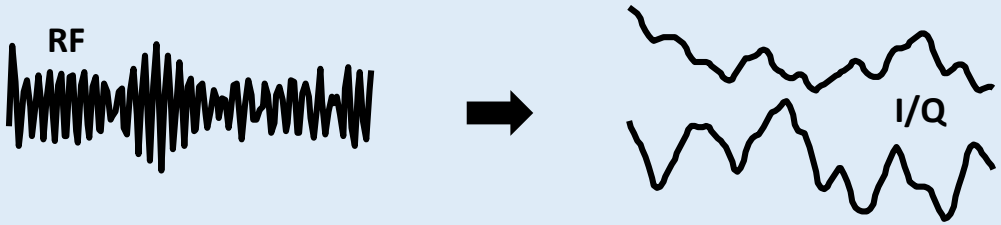


or

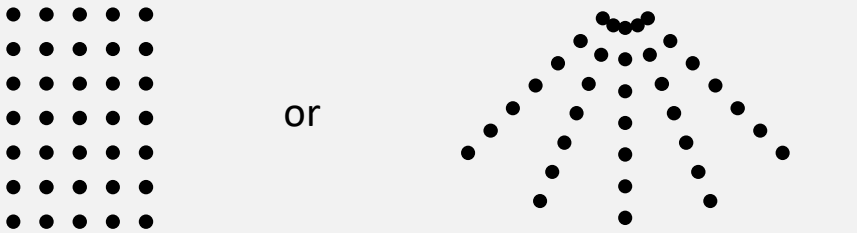


4 steps to generate a color Doppler image

1. Demodulate the received RF signals to obtain I/Q signals



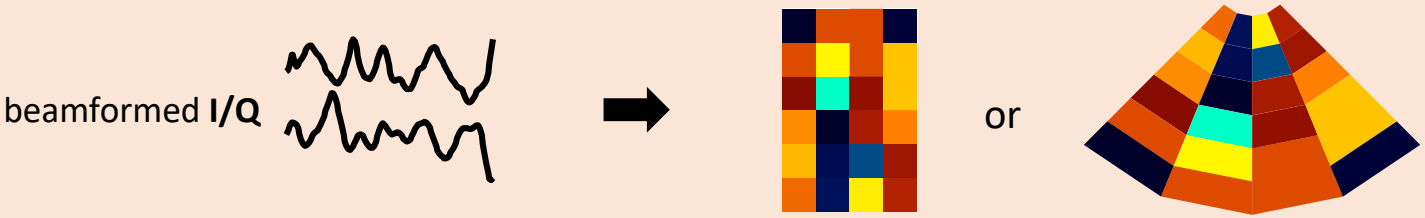
2. Choose a grid for your image



3. Beamform the I/Q signals (e.g. using a delay-and-sum)

```
IQ_beamformed = Mdas*IQ(:);
% Mdas: matrix of size [#grid_points #IQ_samples]
```

4. Use an autocorrelator to obtain a color Doppler image

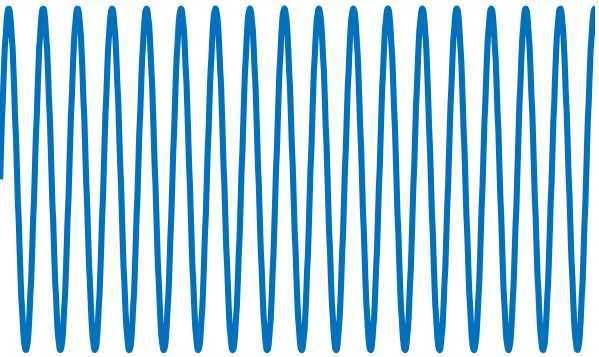


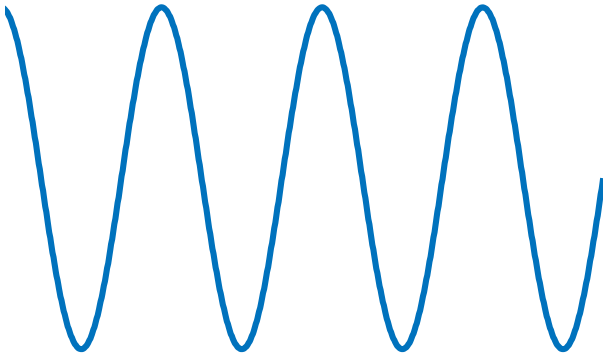
color doppler

Doppler effect: the classical example



© 2009 Christian Wolff





The noise source is approaching
⇒ higher frequency

The noise source is moving away
⇒ lower frequency

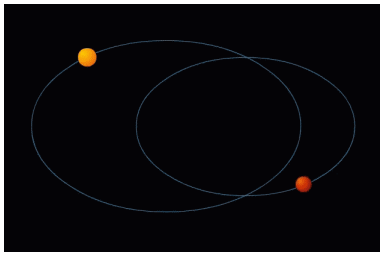
Christian Doppler

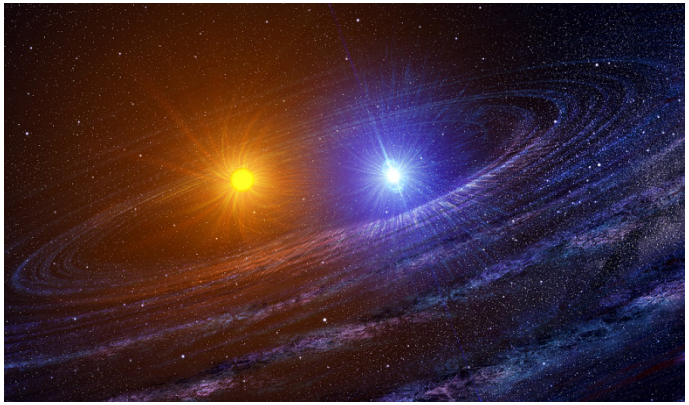
- The observed frequency of a wave depends on the relative speed of the source and the observer
- Christian Doppler explained the changing colors of binary stars

Christian Doppler

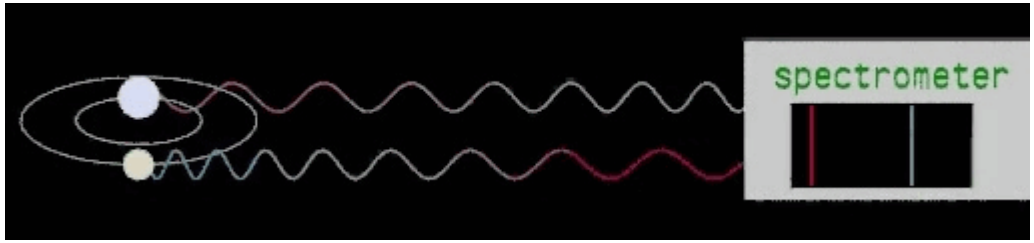


Born	29 November 1803 Salzburg, Austrian Monarchy
Died	17 March 1853 (aged 49) Venice, Kingdom of Lombardy–Venetia
Nationality	Austrian



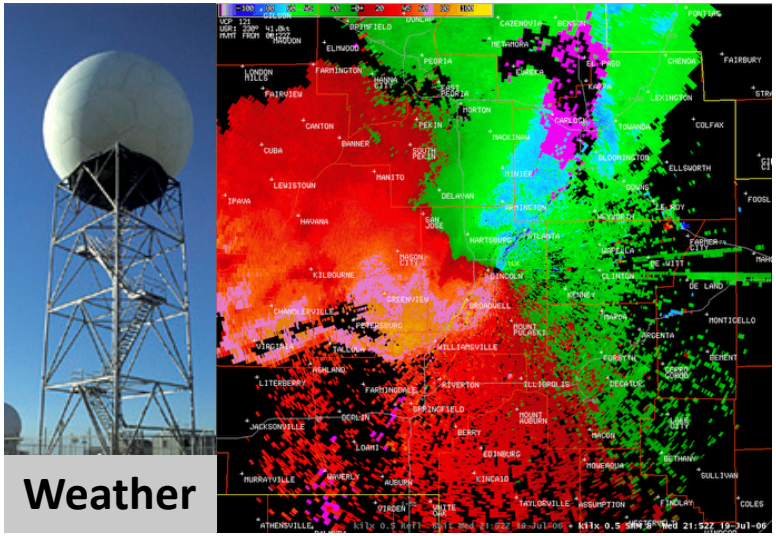


absorption line spectrum

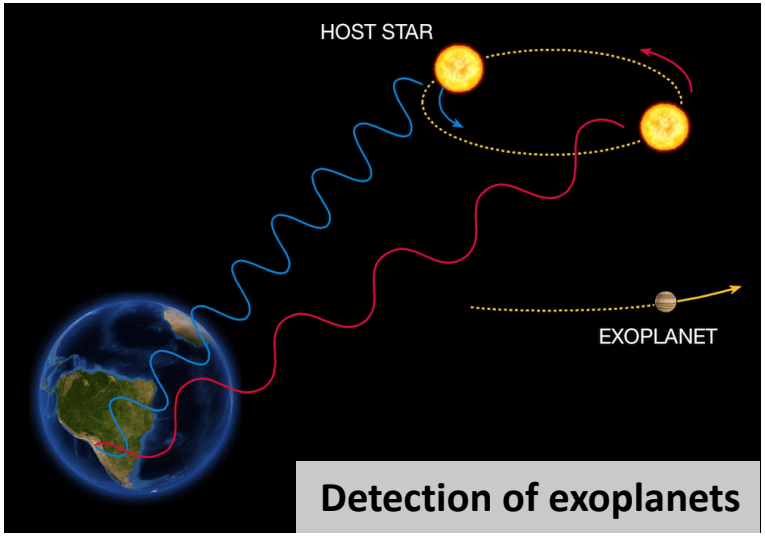


source: *Animations for Physics and Astronomy* (http://phys23p.sl.psu.edu/phys_anim/astro/indexer_astroB.html)

Doppler effect & applications



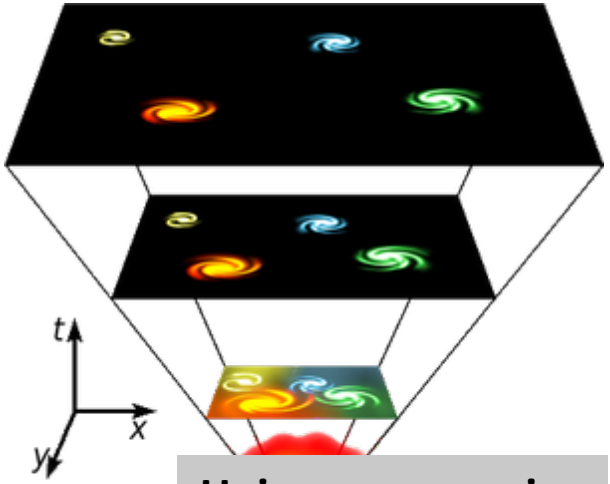
Weather



Detection of exoplanets

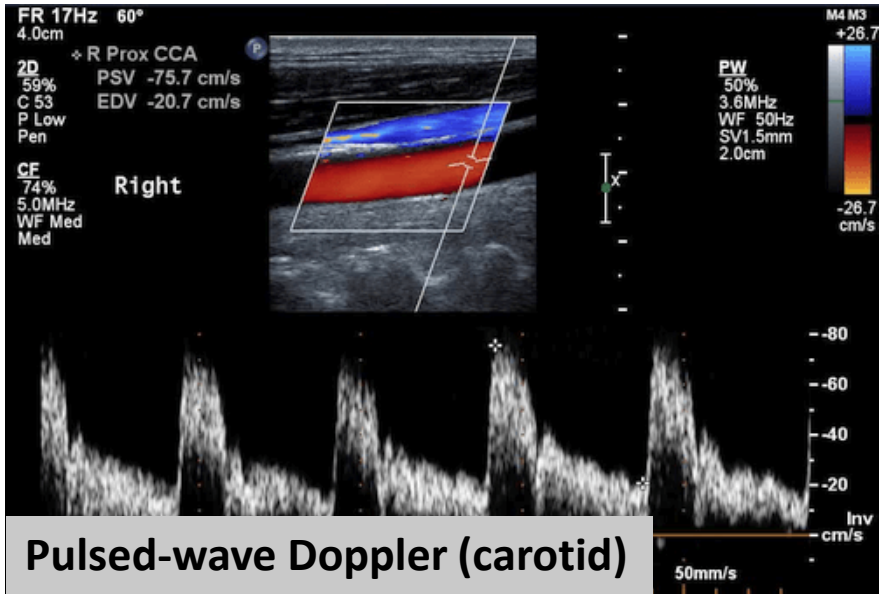


License suspension

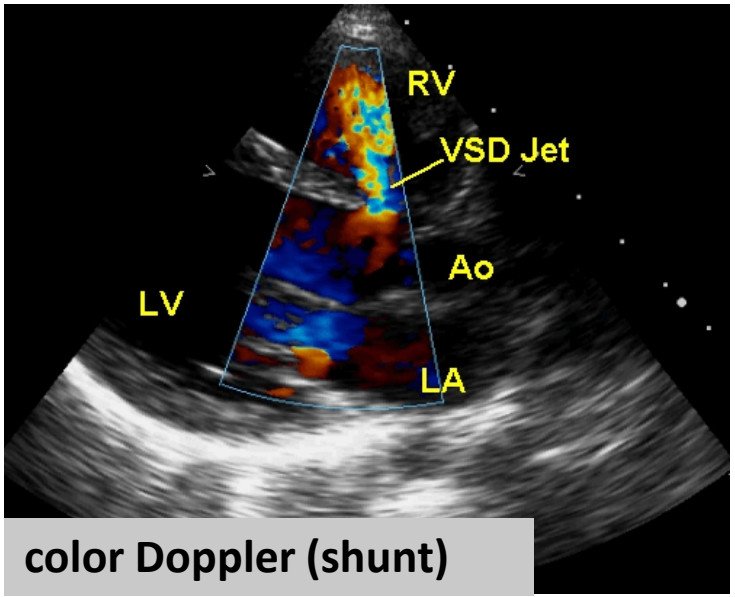


Universe expansion

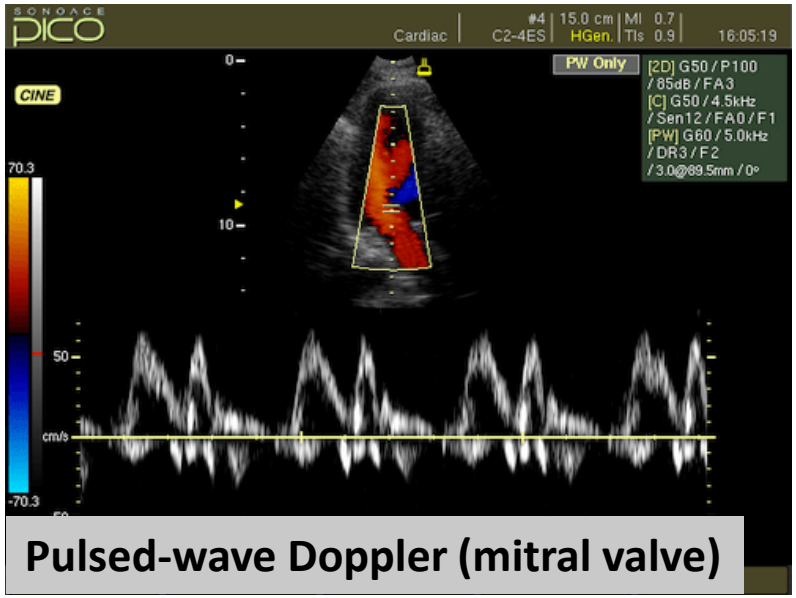
Doppler echocardiography



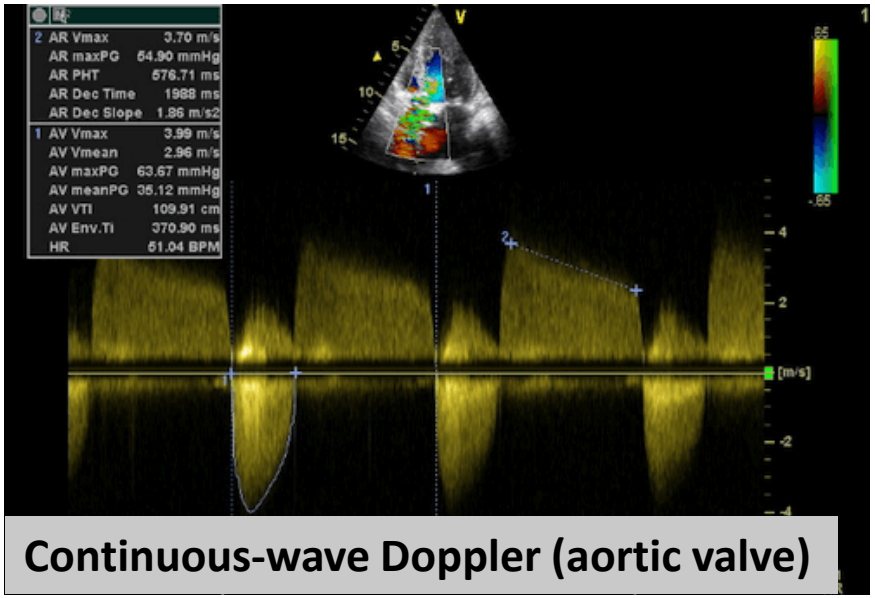
Pulsed-wave Doppler (carotid)



color Doppler (shunt)



Pulsed-wave Doppler (mitral valve)

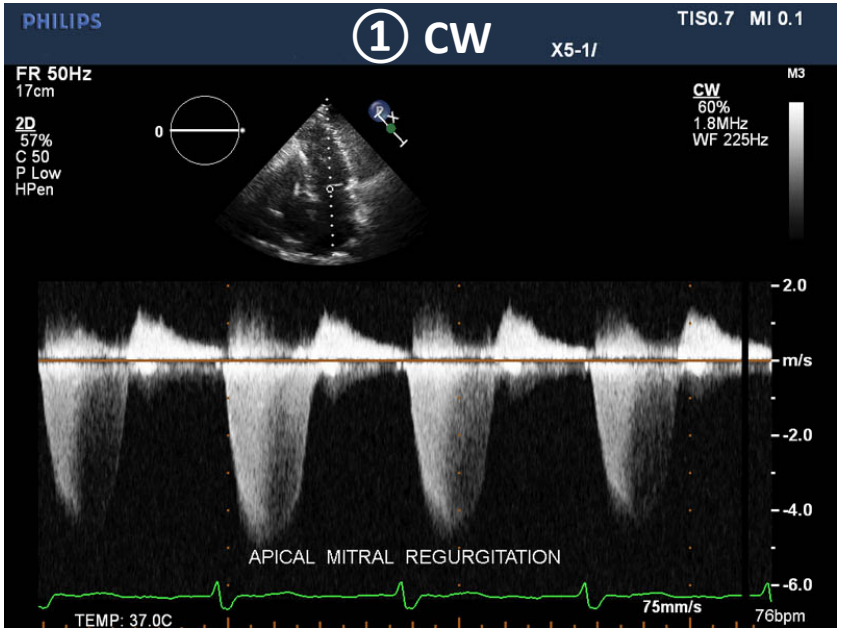


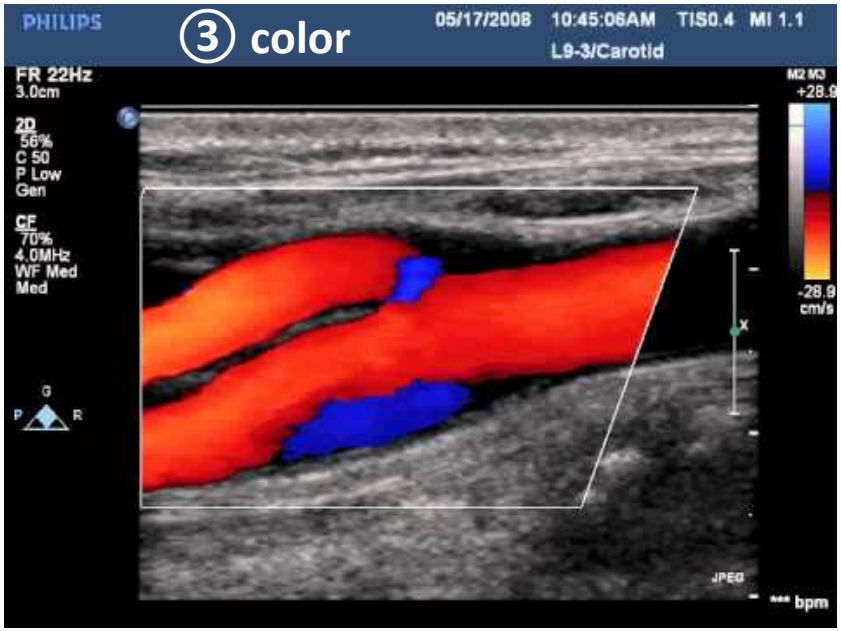
Continuous-wave Doppler (aortic valve)

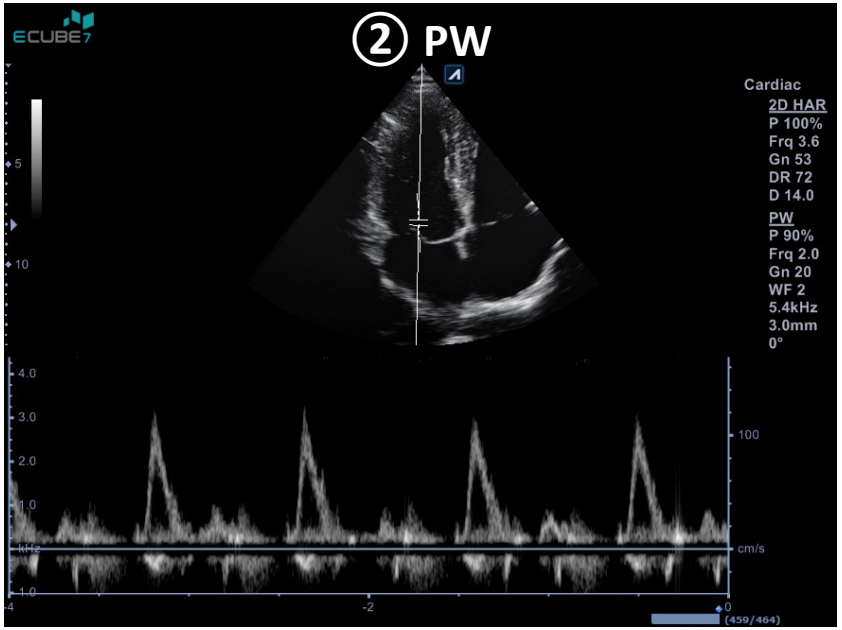
3 main modes in Doppler echo

1. **Continuous-wave Doppler**
2. **Pulsed-wave Doppler**
   (both spectral Doppler)

3. **Color Doppler**



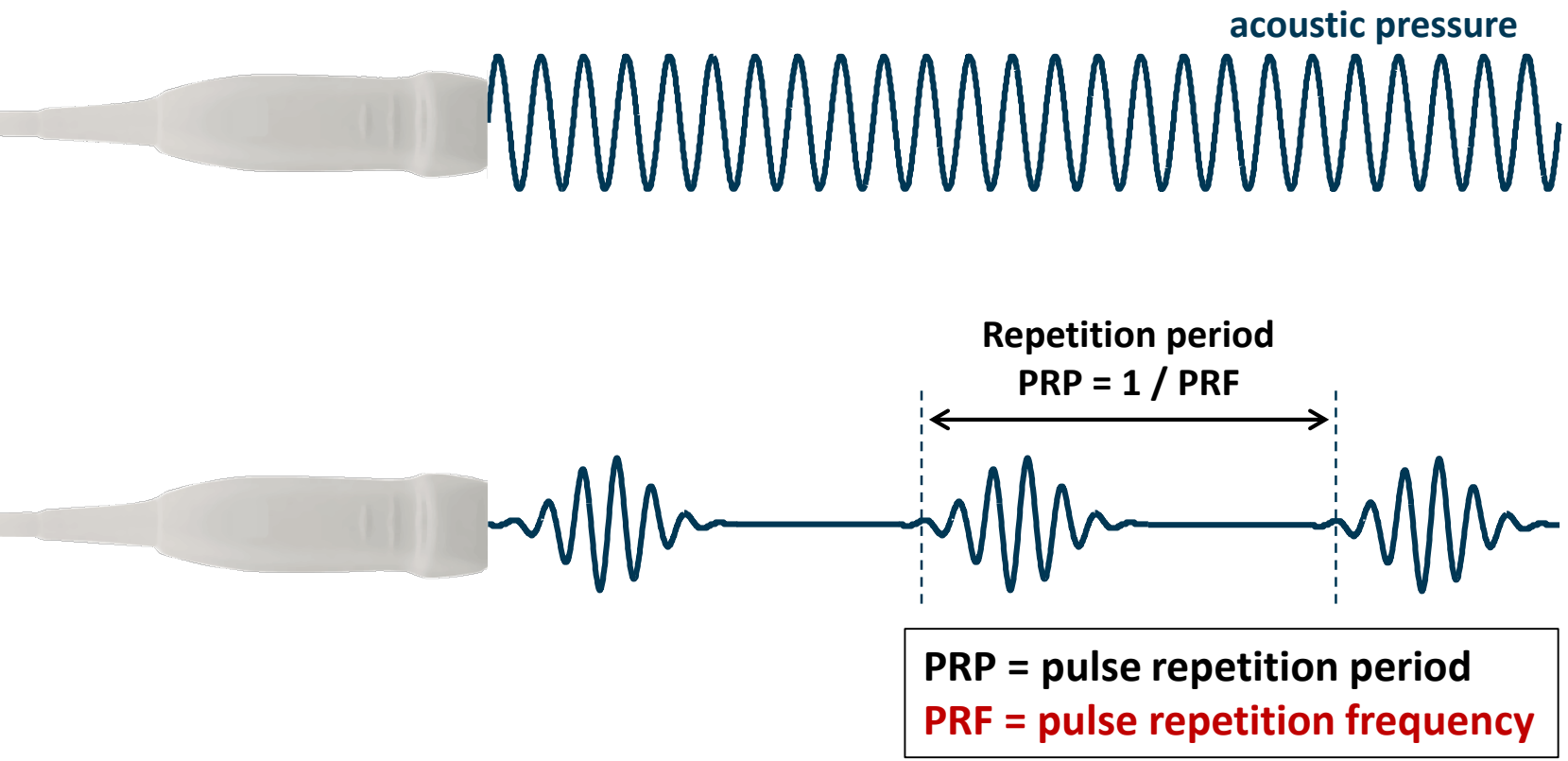




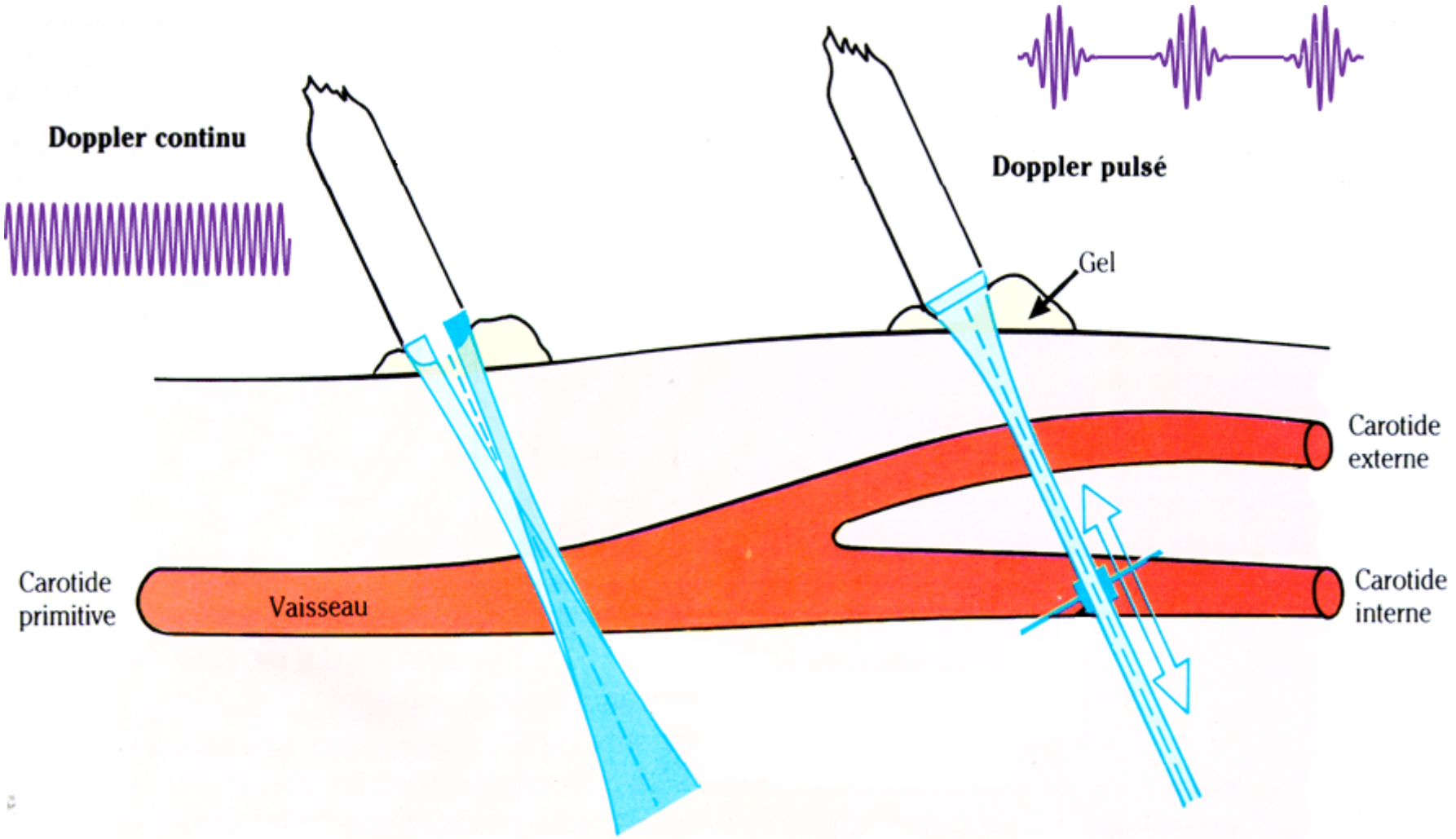
a very brief word about spectral Doppler…

continuous-wave vs. pulsed-wave Doppler

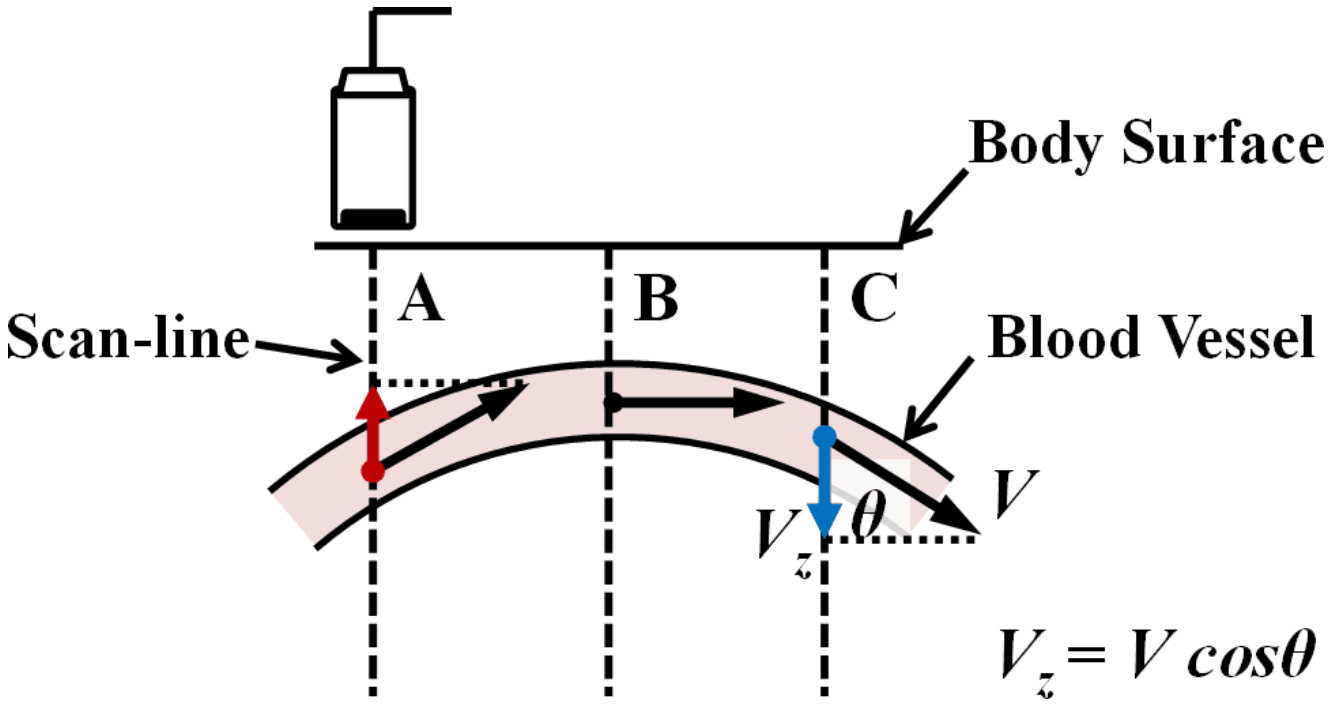
- Two propagation modes are used in spectral Doppler
  1. **continuous-wave Doppler ("CW Doppler")**
  2. **pulsed-wave Doppler ("PW Doppler")**

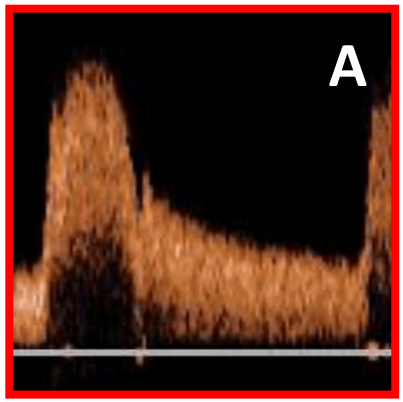


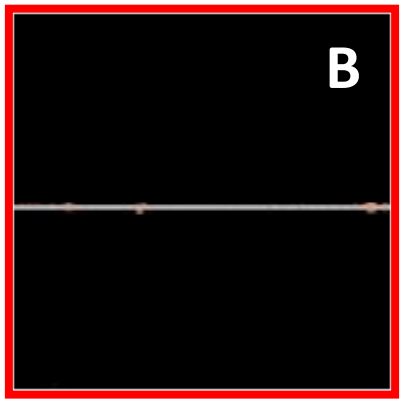
continuous-wave vs. pulsed-wave Doppler

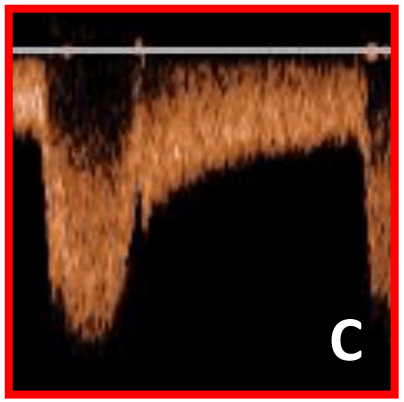


effect of the beam-to-flow angle

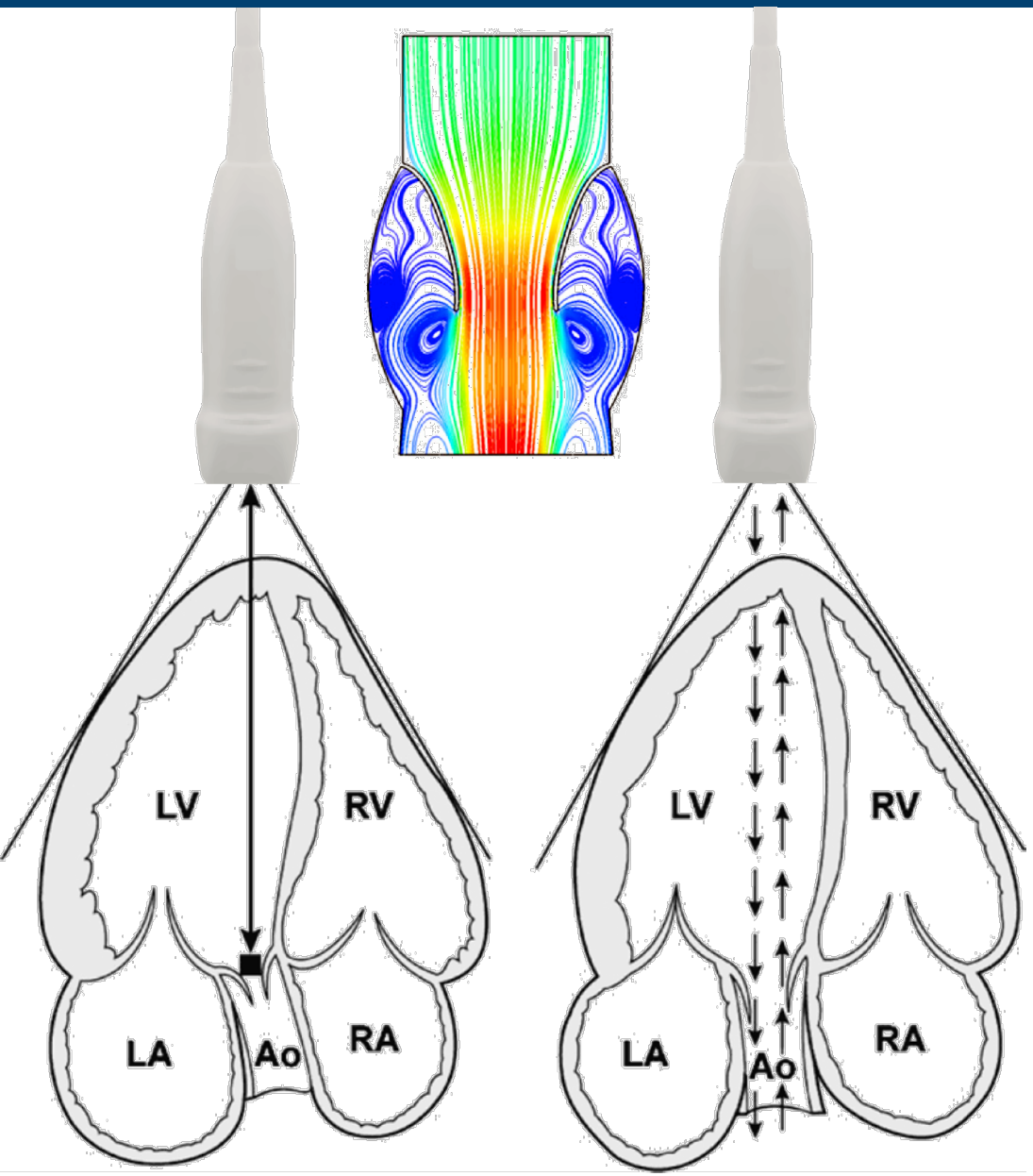




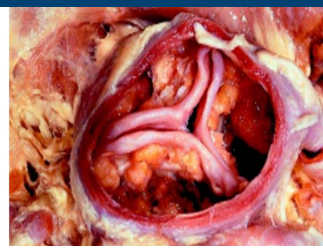


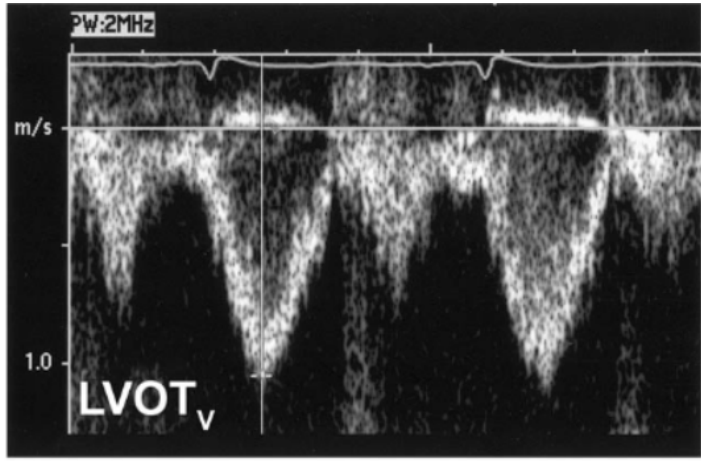


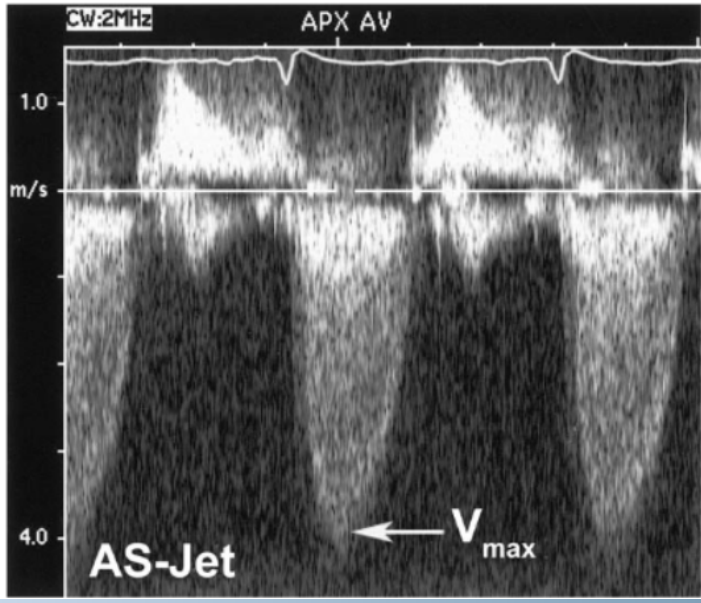
continuous-wave vs. pulsed-wave Doppler









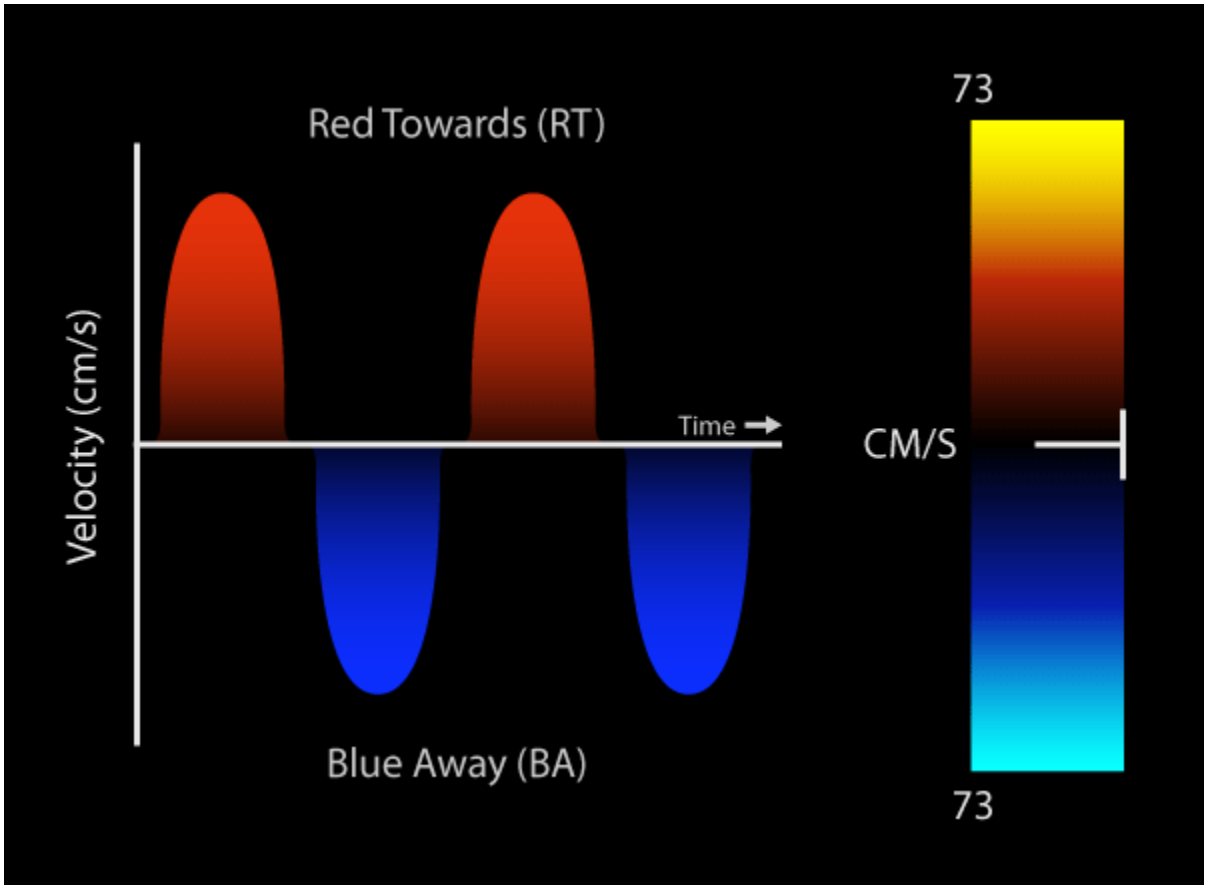


end of…
a very brief word about spectral Doppler

blue-red conventional colors

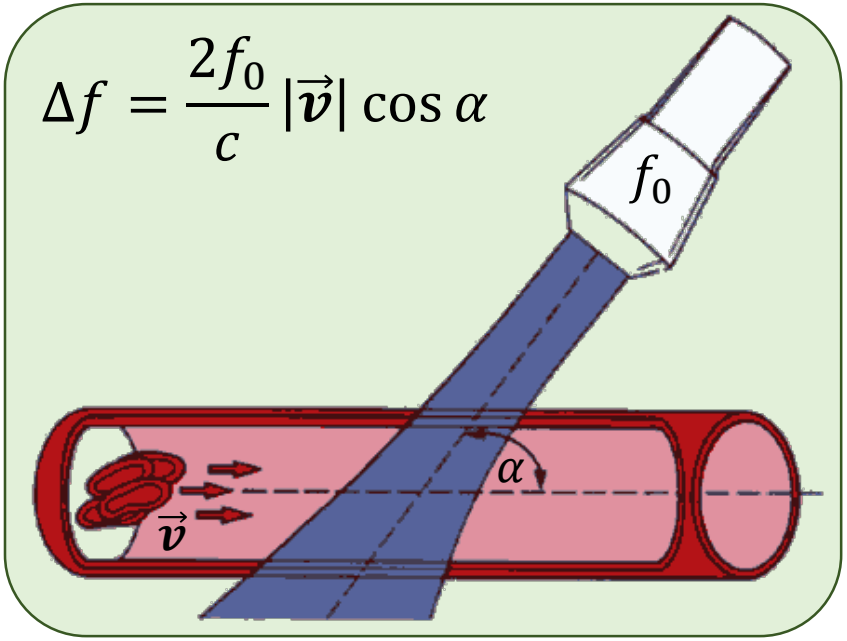
Flow moving towards the probe: RED

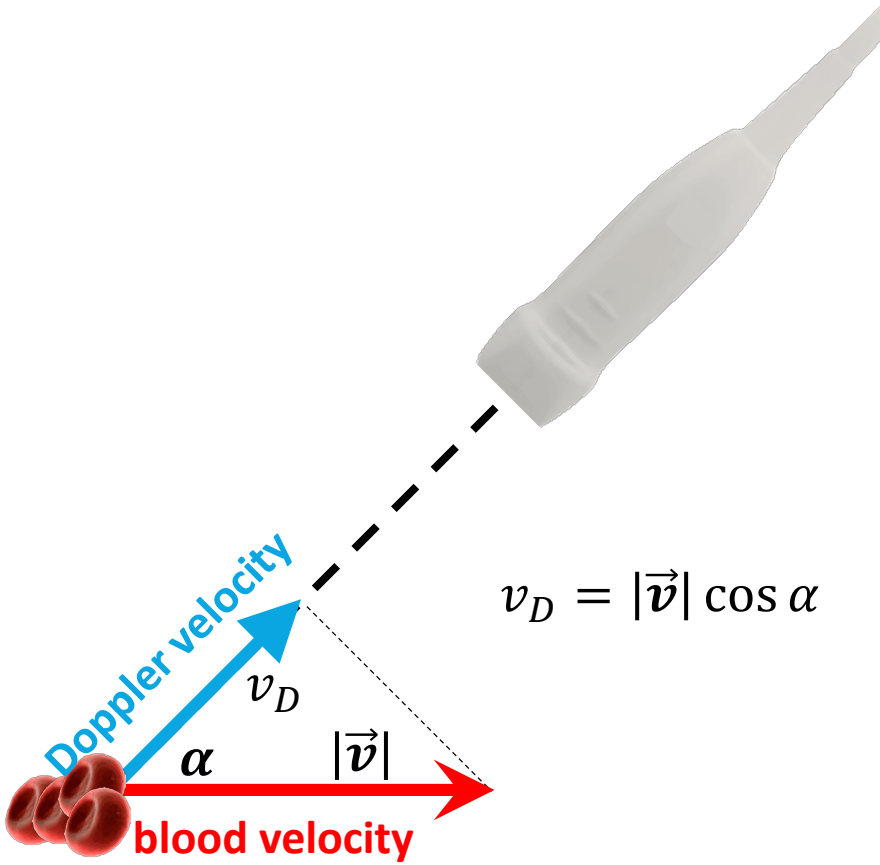
Flow moving away from the probe: BLUE



Doppler echo returns the projection of blood velocity along the propagation axis of the ultrasonic wave

$$\Delta f = \frac{2f_0}{c}|\vec{v}|\cos\alpha$$

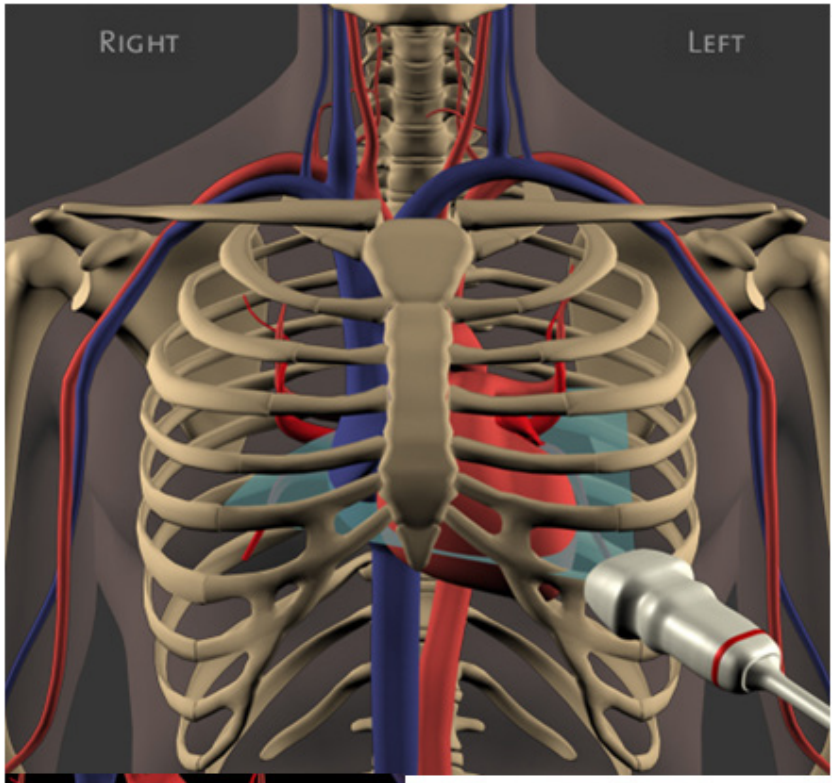


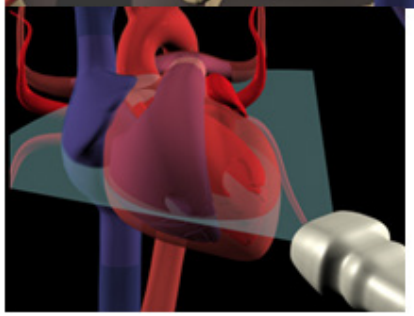


$$v_D = |\vec{v}|\cos\alpha$$

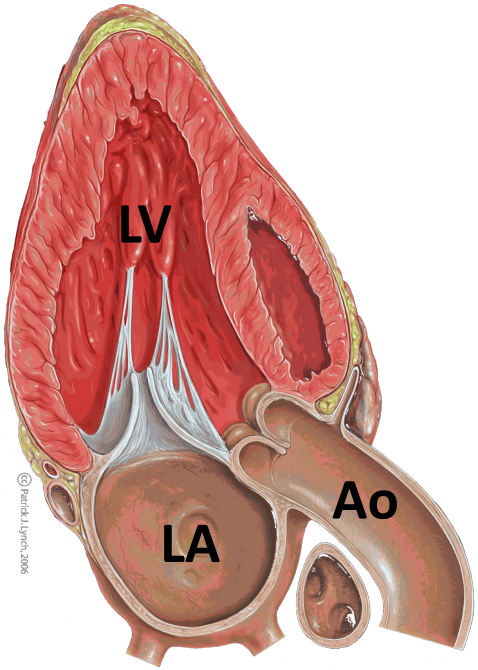
example: Doppler echocardiography

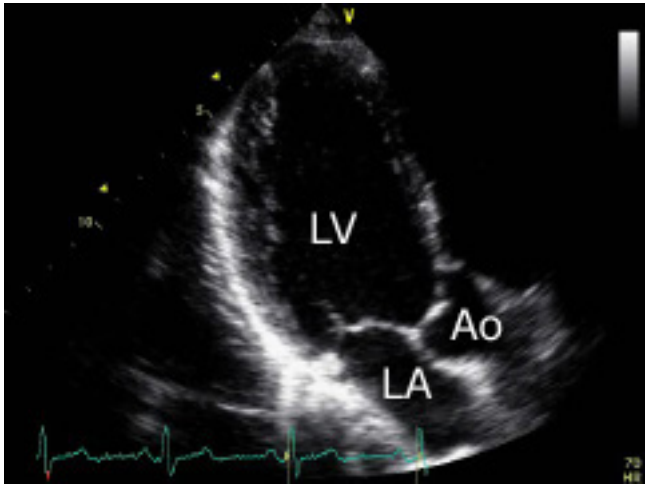




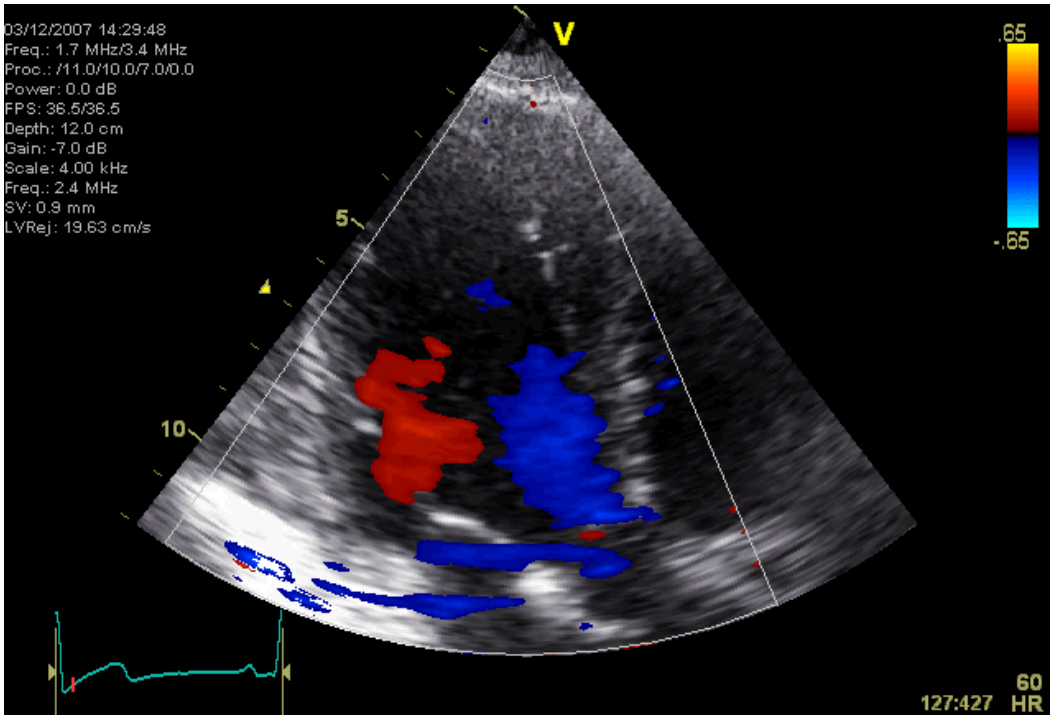


example: Doppler echocardiography

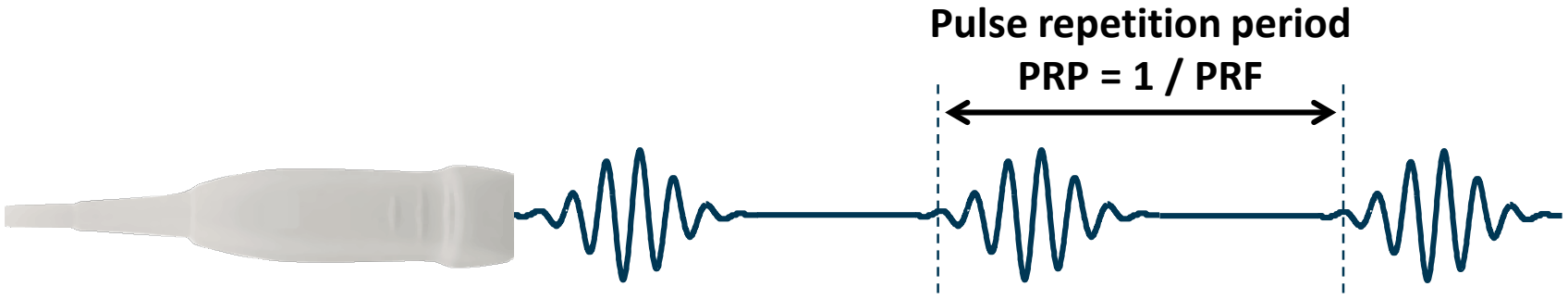




LV : left ventricle
LA : left atrium
Ao : aorta

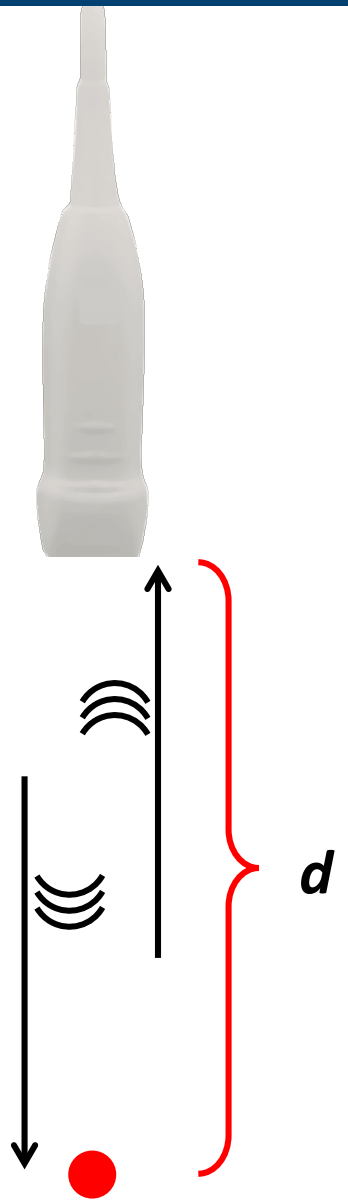


pulse repetition frequency



PRP = pulse repetition period
PRF = pulse repetition frequency

pulse repetition frequency



$$PRF = \frac{c}{2\max(d)}$$

c	wave speed (m/s)
$\max(d)$	maximal depth (m)
PRF	pulse repetition frequency (Hz)

pulse repetition frequency & Nyquist velocity

Doppler equation

$$v_D = \frac{c}{2}\frac{\Delta f}{f_0}$$

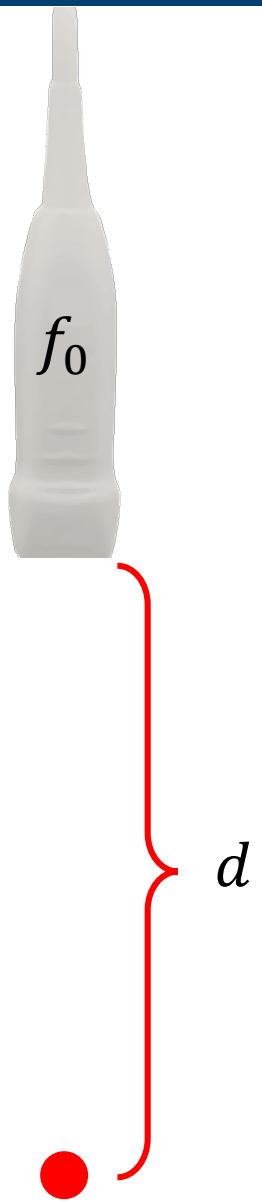
v_D Doppler velocity (m/s)
c wave speed (m/s)
Δf Doppler frequency shift (Hz)
f_0 center frequency (Hz)

Nyquist-Shannon sampling rule ➡ $\max(\Delta f) = \frac{1}{2} PRF$

Nyquist velocity

$$v_N \equiv \max(v_D) = \frac{c\ PRF}{4\ f_0}$$

depth & Nyquist velocity



$$v_N \max(d) = \frac{c^2}{8 f_0} = constant$$

c	wave speed (m/s)
f_0	center frequency (Hz)
$\max(d)$	maximal depth (m)
v_N	**Nyquist velocity** (m/s)

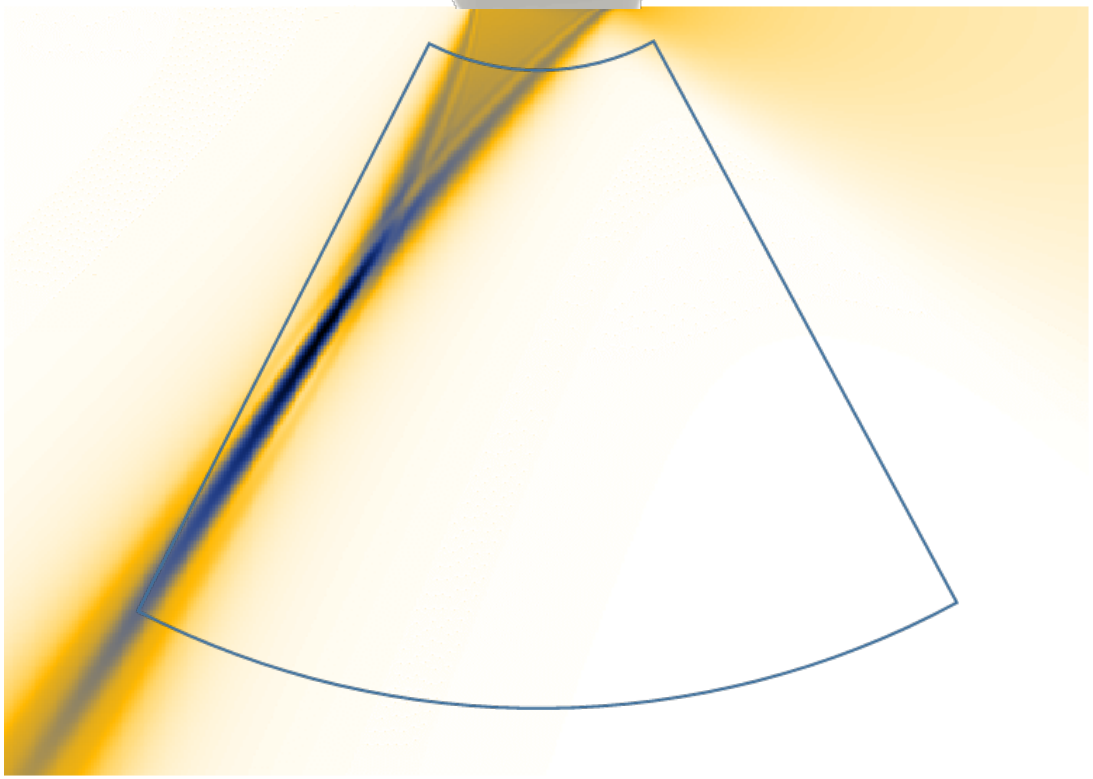
This equation shows the **compromise maximal velocity** *vs*. **maximal depth**

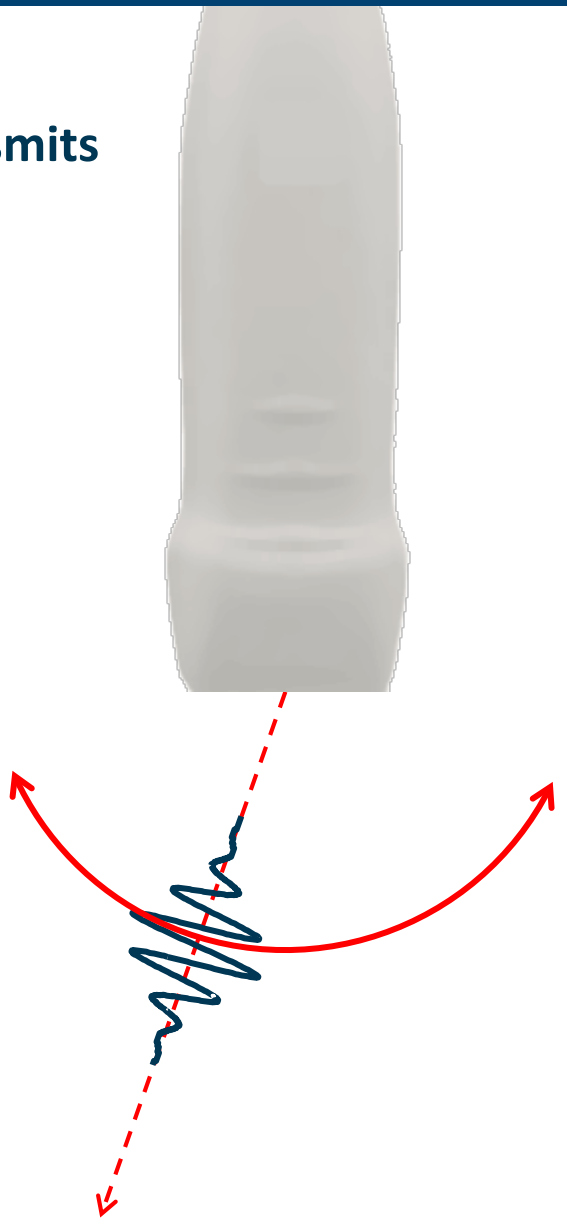
conventional color Doppler

Sweeping + Multiple successive transmits

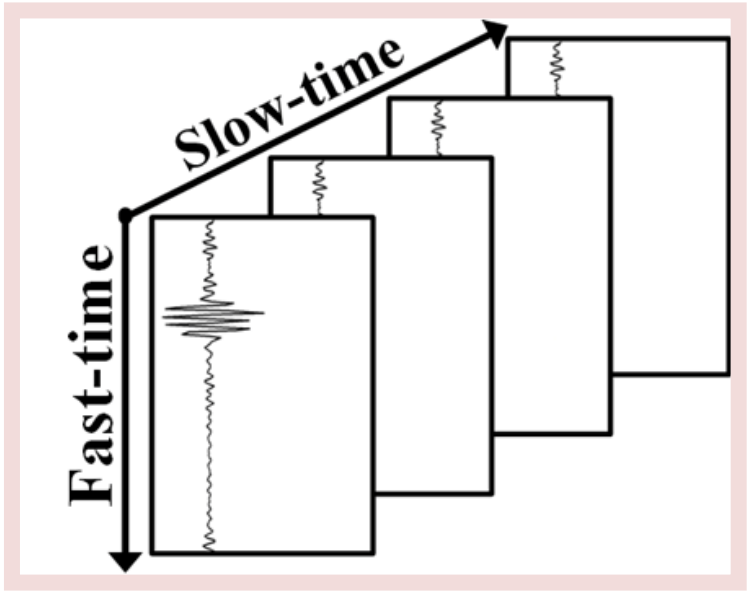
=

Reduced image rate (~10-20 fps)

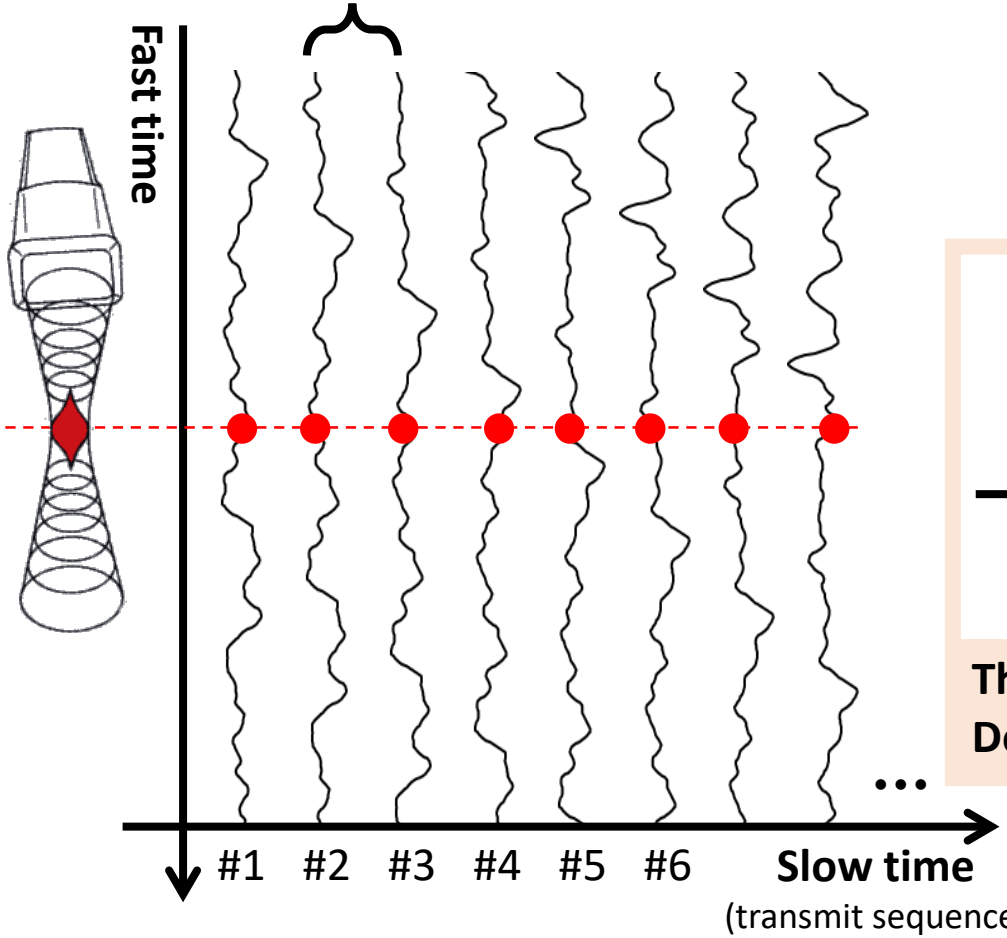


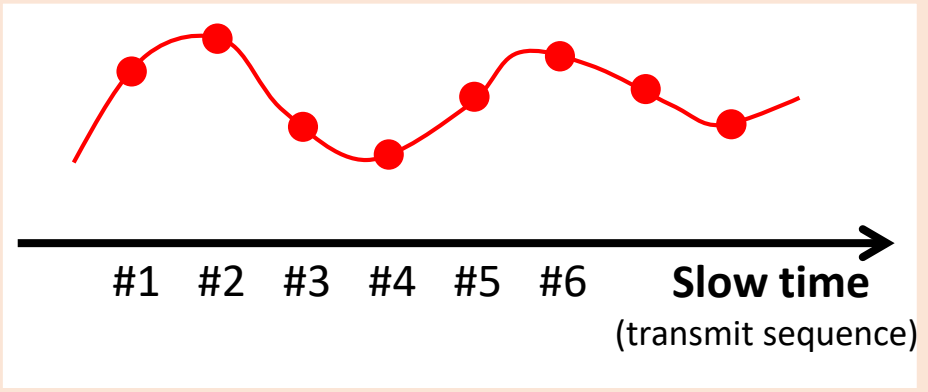


principle of color Doppler



PRP = pulse repetition frequency





The phase shifts of this signal yields the Doppler velocity

in color Doppler, Doppler velocities are generally estimated through temporal **auto-correlation of the I/Q signals**

- **Doppler velocity by I/Q autocorrelation – Proof**

In-phase and quadrature signals:

$$\begin{cases} I(x,z,\tau) = A(x,z,\tau)\cos\big(\varphi(x,z,\tau)\big) \\ Q(x,z,\tau) = A(x,z,\tau)\sin\big(\varphi(x,z,\tau)\big) \end{cases}$$

We define the complex I/Q signal by:

$$IQ(\tau) \equiv I(\tau) + iQ(\tau) = A(\tau)e^{i\varphi(\tau)}$$

Its numerical counterpart is

$$IQ_k = A_k e^{i\varphi_k}$$

And the conjugate is

$$\overline{IQ_k} = A_k e^{-i\varphi_k}$$

…

- **Doppler velocity by I/Q autocorrelation – <u>Proof</u>** (cont'd)

...

The auto-correlation of lag 1 is

$$\mathcal{R}_1 = \sum_{k=1}^{N-1} IQ_k \, \overline{IQ_{k+1}} = \sum_{k=1}^{N-1} A_k A_{k+1} \, e^{i(\varphi_k - \varphi_{k+1})}$$

For a given packet of length N, we assume that:

$$\varphi_{k+1} - \varphi_k = \Delta\varphi = \text{constant}, \forall k \in [1, N]$$

The auto-correlation thus can read:

$$\mathcal{R}_1 = \underbrace{\left[\sum_{k=1}^{N-1} A_k A_{k+1}\right]}_{\alpha} e^{-i\Delta\varphi} = \alpha e^{-i\Delta\varphi}$$

Using the Napierian logarithm, we obtain

$$\ln(\mathcal{R}_1) = \ln(\alpha) + \ln\left(e^{-i\Delta\varphi}\right) = \ln(\alpha) - i\Delta\varphi$$

...

- **Doppler velocity by I/Q autocorrelation – Proof** (cont'd and end)

...

The phase-shift can be thus written as

$$\Delta\varphi = -\mathrm{Im}\{\ln(\mathcal{R}_1)\}$$

We recall the Doppler equation: $v_D = \frac{c}{2}\frac{\Delta f}{f_0}$

Because phase-shift is related to frequency-shift by

$$\Delta\varphi = \Delta\omega\Delta\tau = 2\pi\,\Delta f\Delta\tau$$

it follows that the Doppler velocity is given by

$$v_D = \frac{c}{2}\frac{\Delta\varphi}{2\pi\Delta\tau f_0}$$

By definition, the pulse repetition frequency is $PRF \equiv 1/\Delta\tau$

We finally obtain the estimated Doppler velocity:

$$\boxed{v_D = -\frac{c\,PRF}{4\pi f_0}\mathrm{Im}\{\ln(\mathcal{R}_1)\}}$$

- **Doppler velocity by I/Q autocorrelation – a 5-line Matlab code**



```
%-- IQ = I/Q ultrasound signals
% IQ is a 3-D array:
% dim #1-2 = space; dim #3 = slow-time
IQ1 = IQ(:,:,1:end-1);
IQ2 = IQ(:,:,2:end); % lag-1 temporal shift

%-- lag-1 I/Q auto-correlation
R1 = sum(IQ1.*conj(IQ2),3);

%-- Doppler velocity (Vd)
% PRF = pulse repetition frequency
% f0 = center frequency
VN = c*PRF/4/f0; % Nyquist velocity
Vd = -VN*imag(log(R1))/pi;
```

- **Aliasing**

The Doppler velocity can be written as:

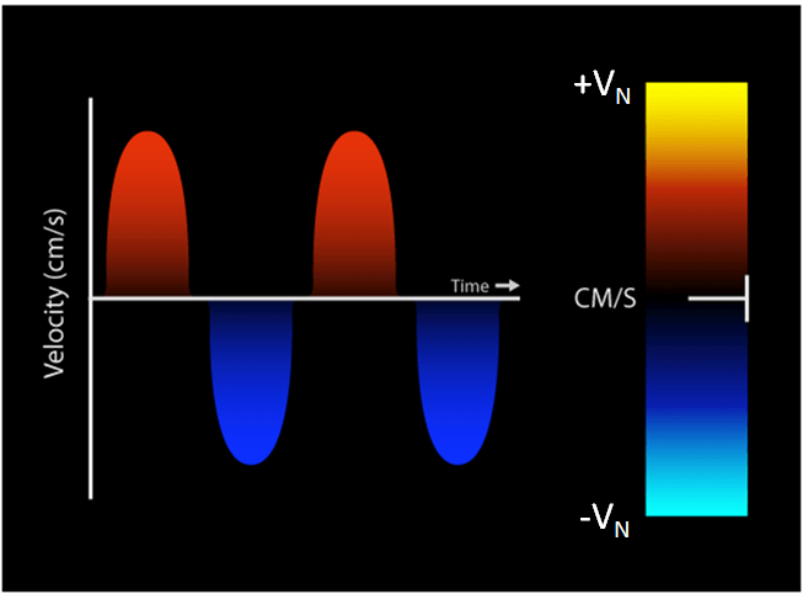
$$v_D = -\underbrace{\left(\frac{c\ PRF}{4\ f_0}\right)}_{v_{\mathrm{N}}} \frac{\mathrm{Im}\{\log(\mathcal{R}_1)\}}{\pi}$$

$\mathrm{Im}\{\log(\mathcal{R}_1)\} = \mathrm{atan2}\{\mathrm{Im}(\mathcal{R}_1), \mathrm{Re}(\mathcal{R}_1)\}$ is included in $[-\pi, +\pi]$.

It follows that v_D is included in $[-v_{\mathrm{N}}, +v_N]$.

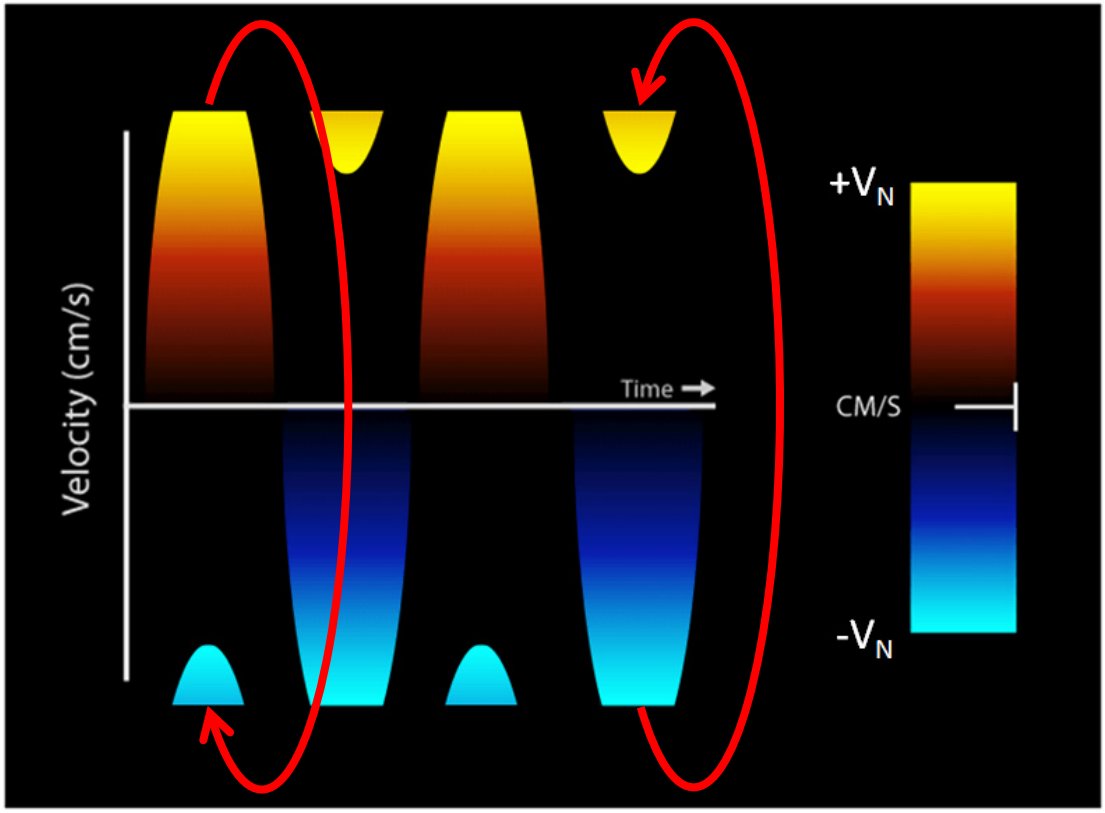
- If the actual velocity is so that $|\vec{v}| > v_{\mathrm{N}}$, there is **aliasing**.
- The largest unambiguous Doppler range is $(2v_{\mathrm{N}})$.

aliasing

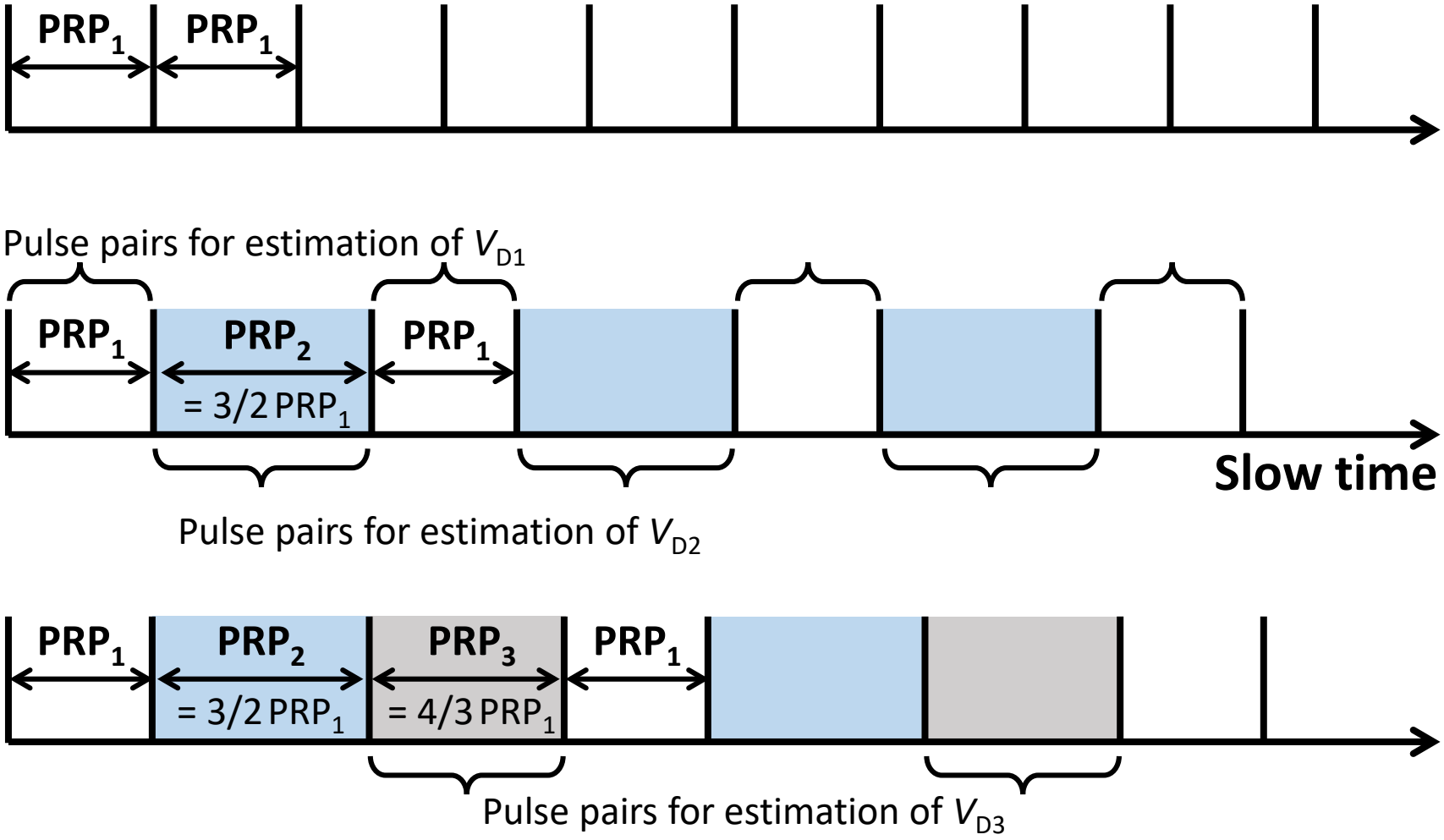


No aliasing

Aliasing

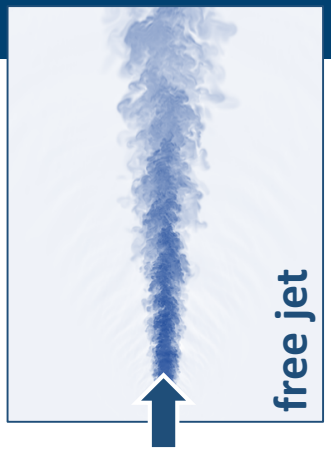


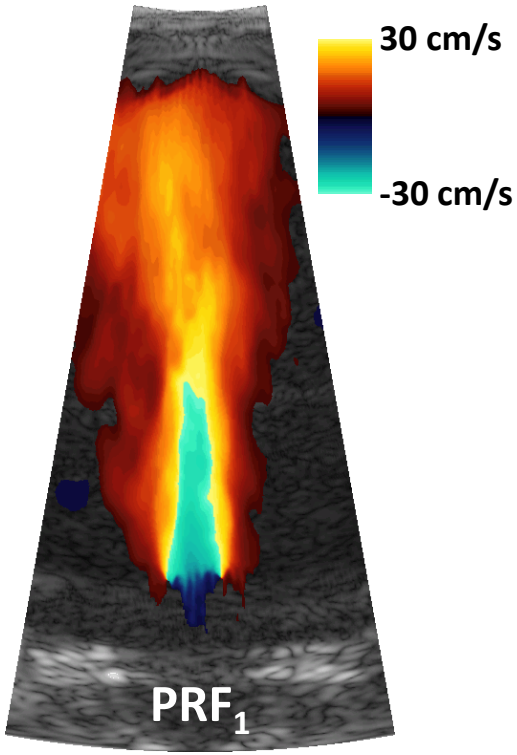
multi-PRF

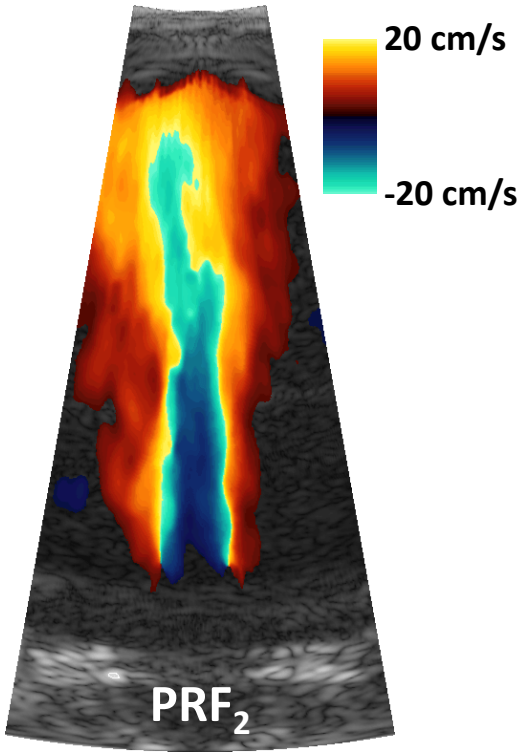


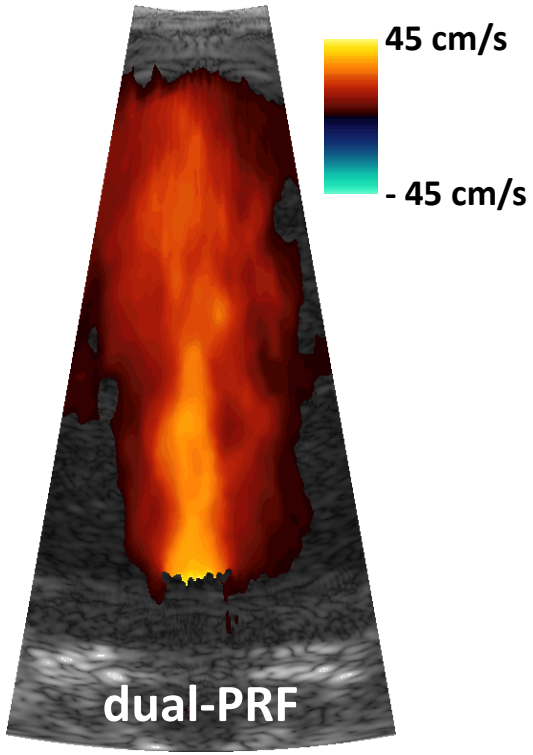
PRP = pulse repetition period
PRF = 1/PRP = pulse repetition frequency

aliasing

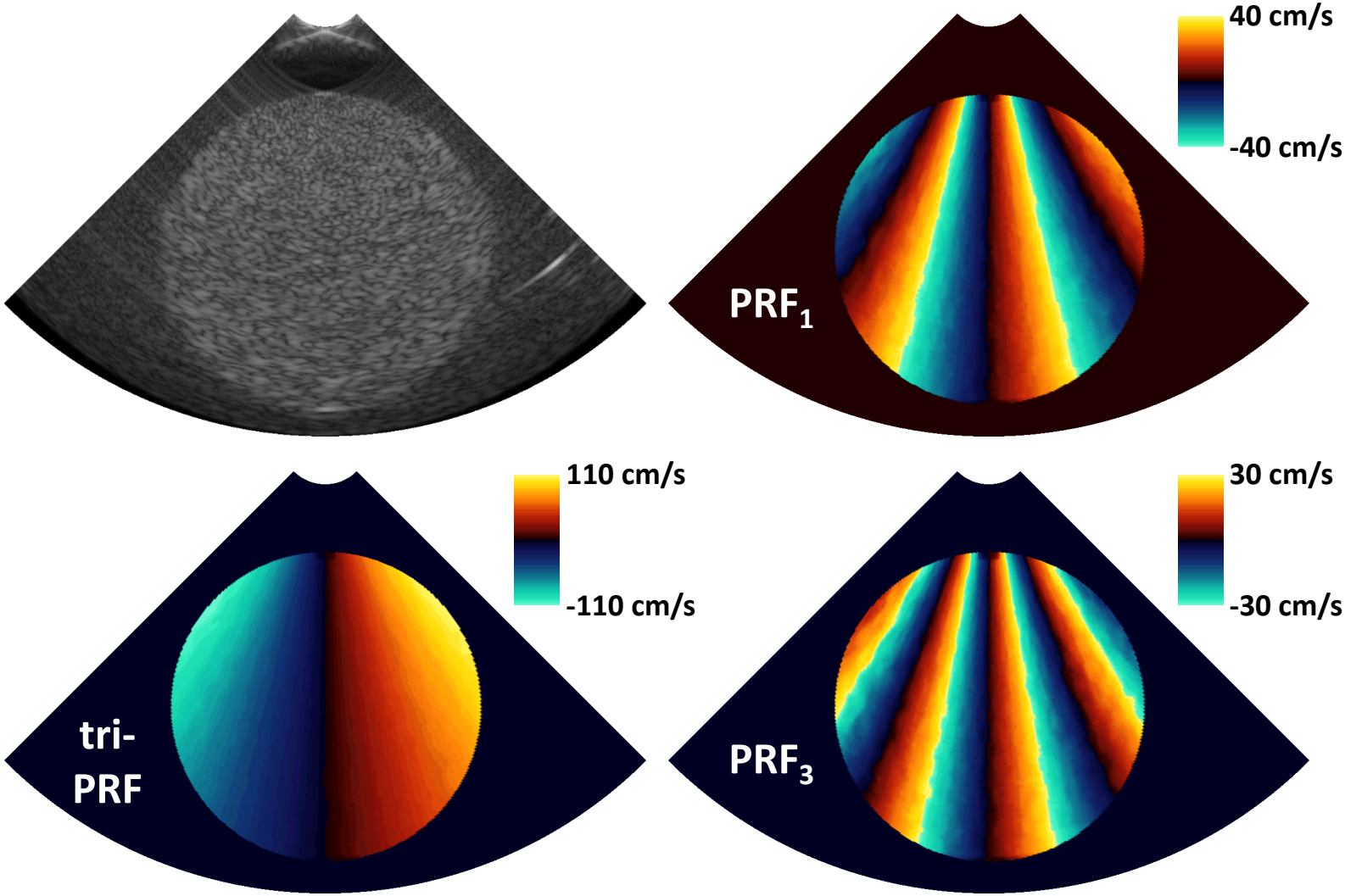

free jet


30 cm/s
-30 cm/s
PRF1

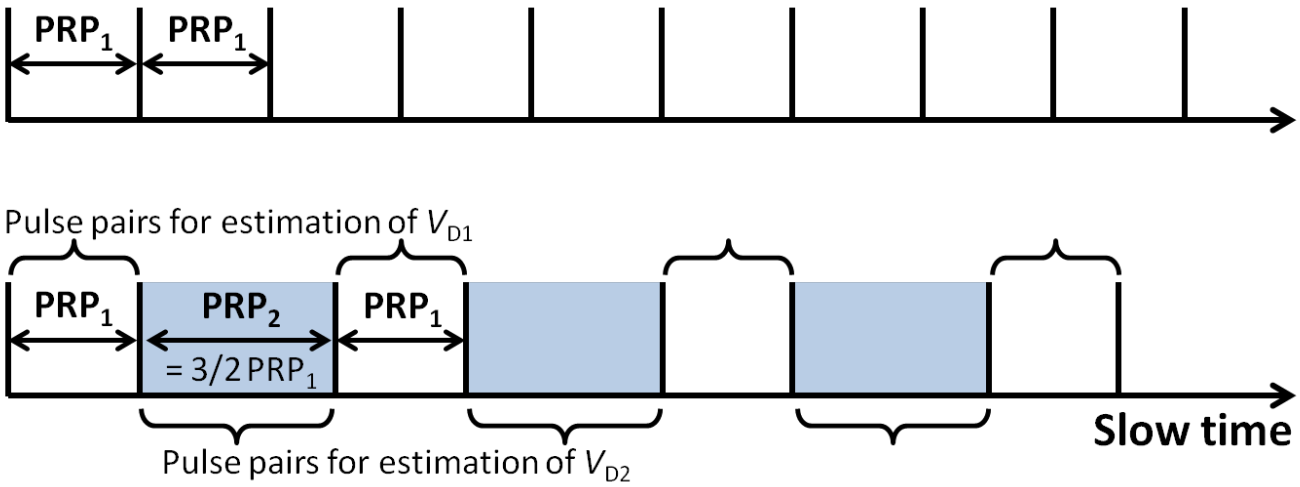

20 cm/s
-20 cm/s
PRF2


45 cm/s
- 45 cm/s
dual-PRF

multi-PRF in a spinning disk



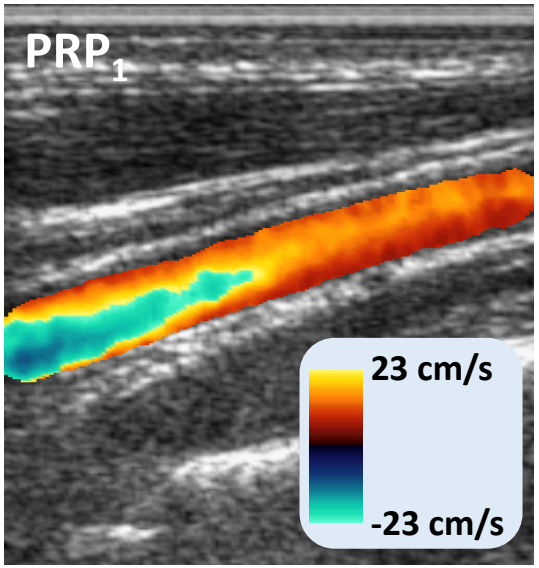
alias-free color Doppler

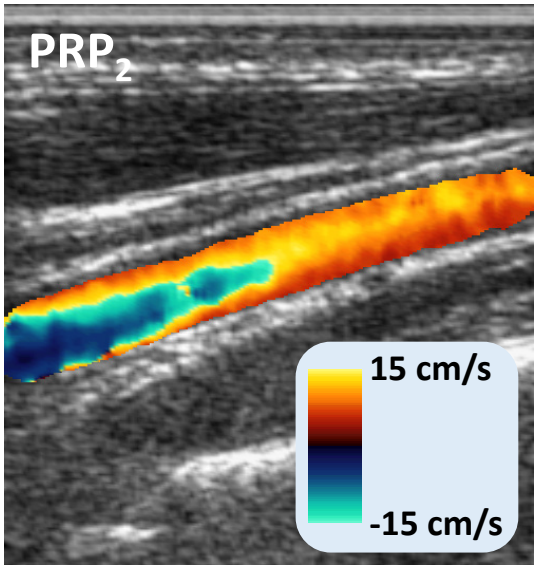


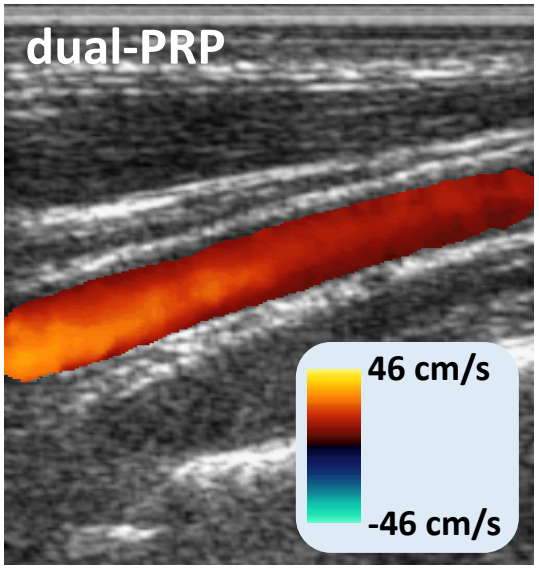
Aliasing can be removed by using multiple Pulse Repetition Periods (PRP)

key paper

- Posada *et al*. IEEE TMI 2016;35:1510-1521



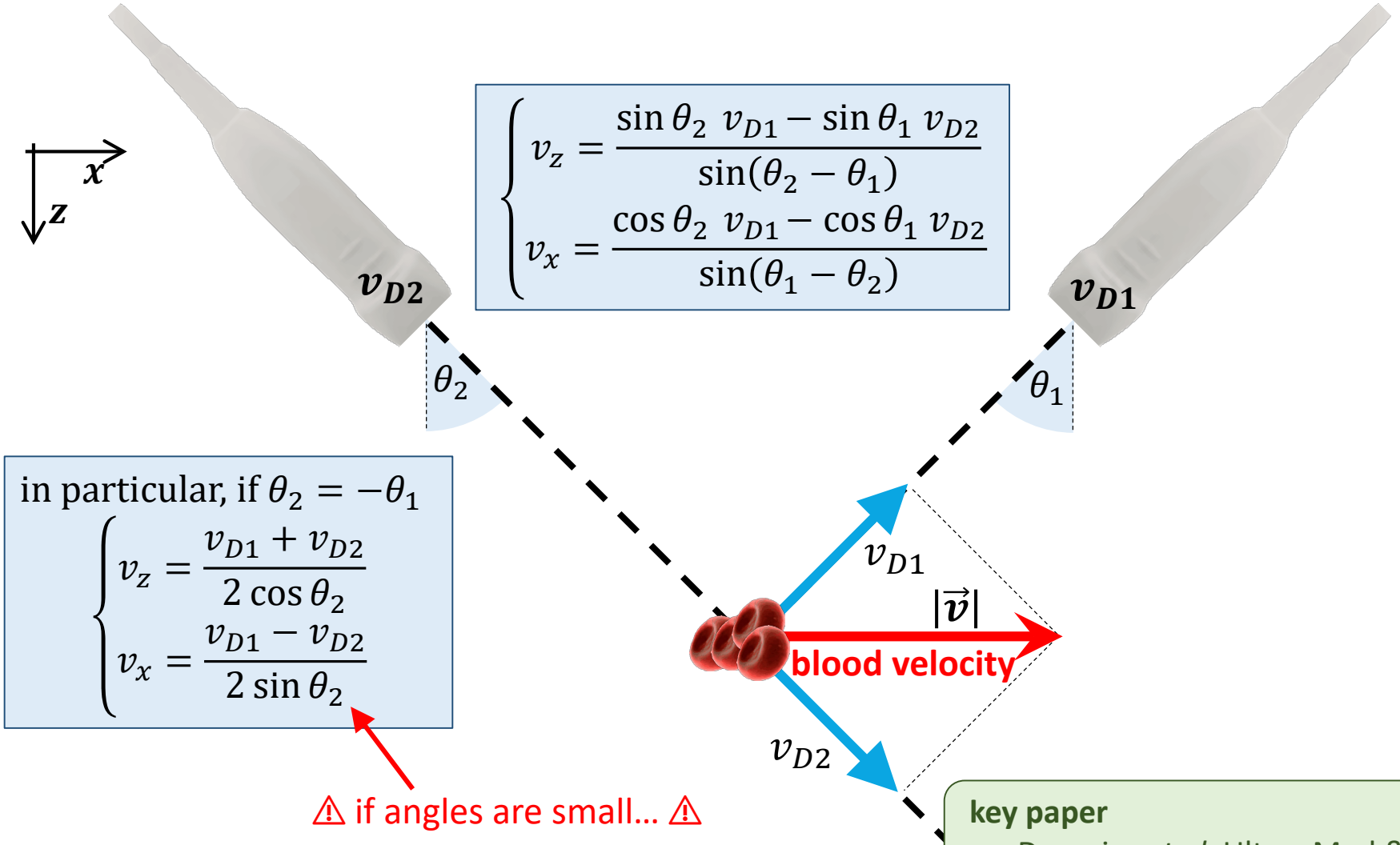




vector doppler

Doppler echo returns a projection of blood velocity vector...

IDEA: use 2 (in 2-D) or 3 (in 3-D) projections to get complete information!



$$\begin{cases} v_z = \dfrac{\sin\theta_2\ v_{D1} - \sin\theta_1\ v_{D2}}{\sin(\theta_2 - \theta_1)} \\ v_x = \dfrac{\cos\theta_2\ v_{D1} - \cos\theta_1\ v_{D2}}{\sin(\theta_1 - \theta_2)} \end{cases}$$

in particular, if $\theta_2 = -\theta_1$

$$\begin{cases} v_z = \dfrac{v_{D1} + v_{D2}}{2\cos\theta_2} \\ v_x = \dfrac{v_{D1} - v_{D2}}{2\sin\theta_2} \end{cases}$$

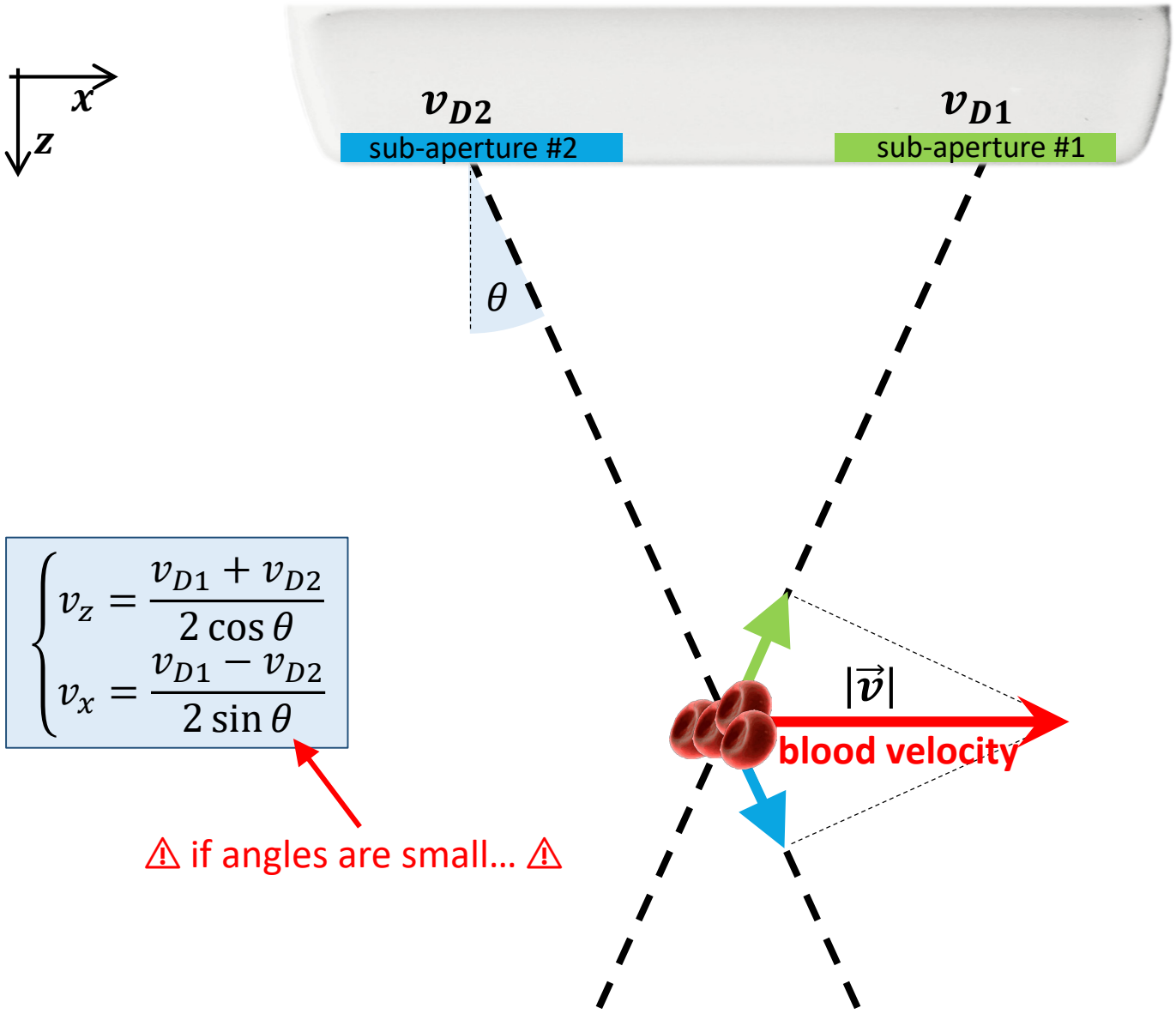
⚠ if angles are small... ⚠

key paper

- Dunmire *et al*. Ultras Med & Biol 2000;26:1213-1235

X-beam vector Doppler using two subapertures

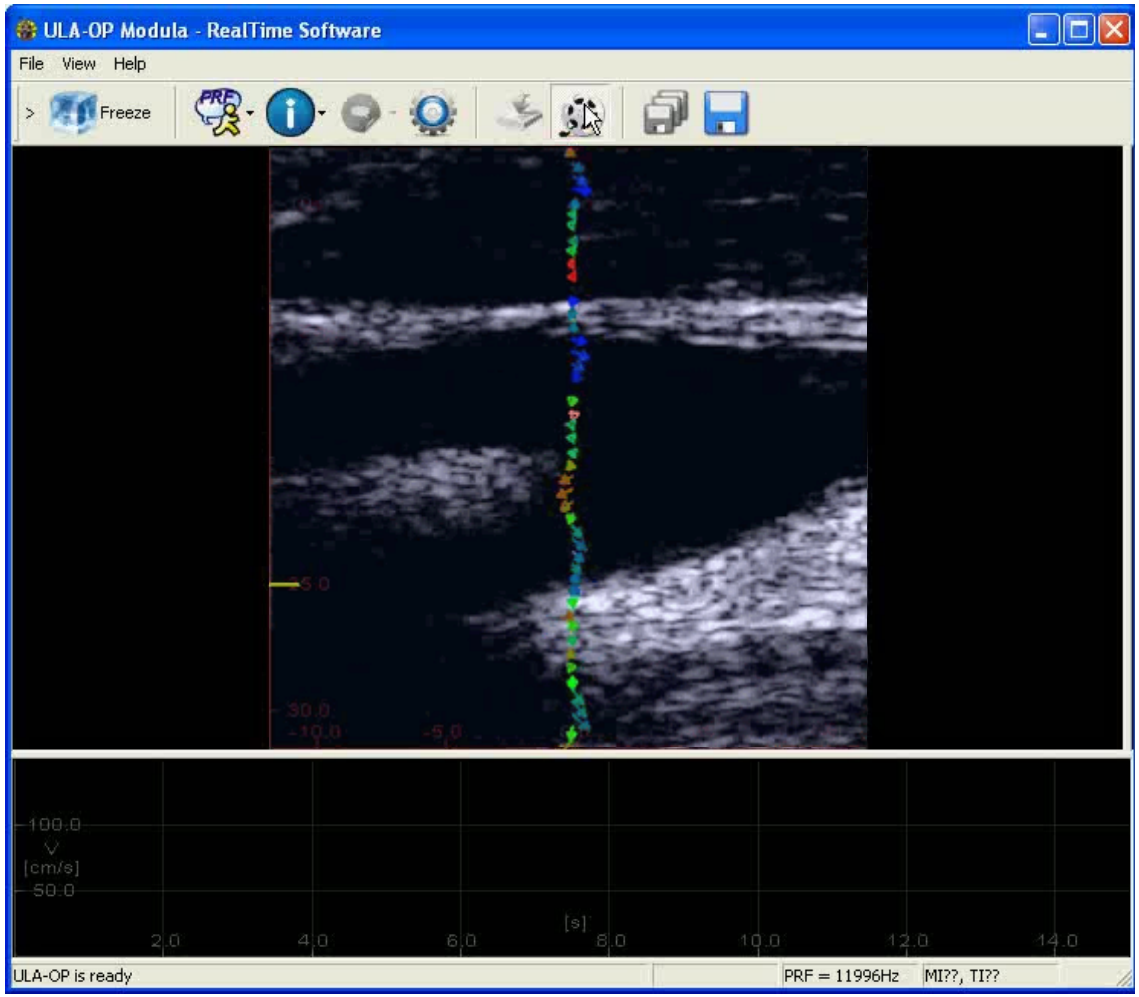
Use two successive sub-apertures instead of using two transducers…

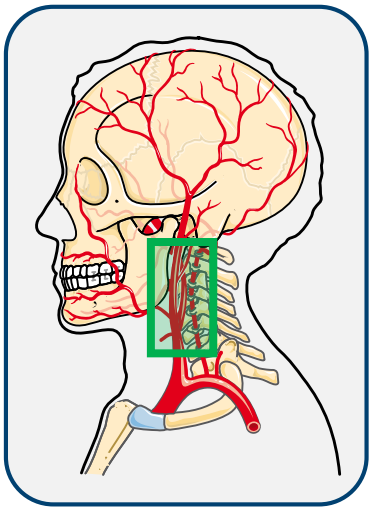


$$\begin{cases} v_z = \dfrac{v_{D1} + v_{D2}}{2\cos\theta} \\ v_x = \dfrac{v_{D1} - v_{D2}}{2\sin\theta} \end{cases}$$

⚠ if angles are small… ⚠

X-beam vector Doppler w. two subapertures in real time





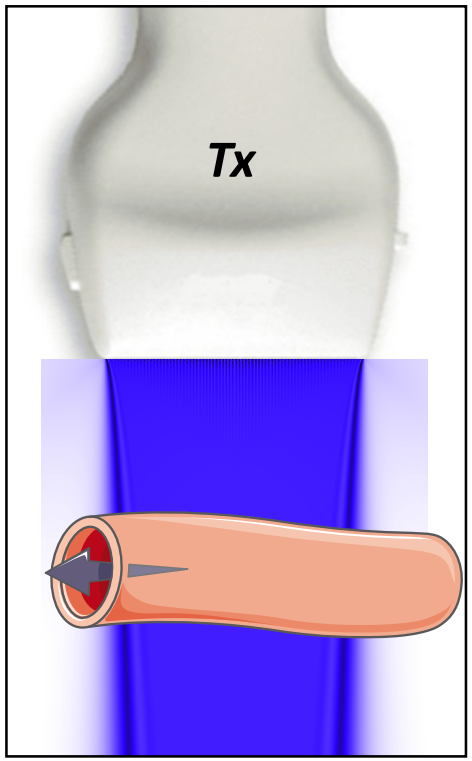


key paper

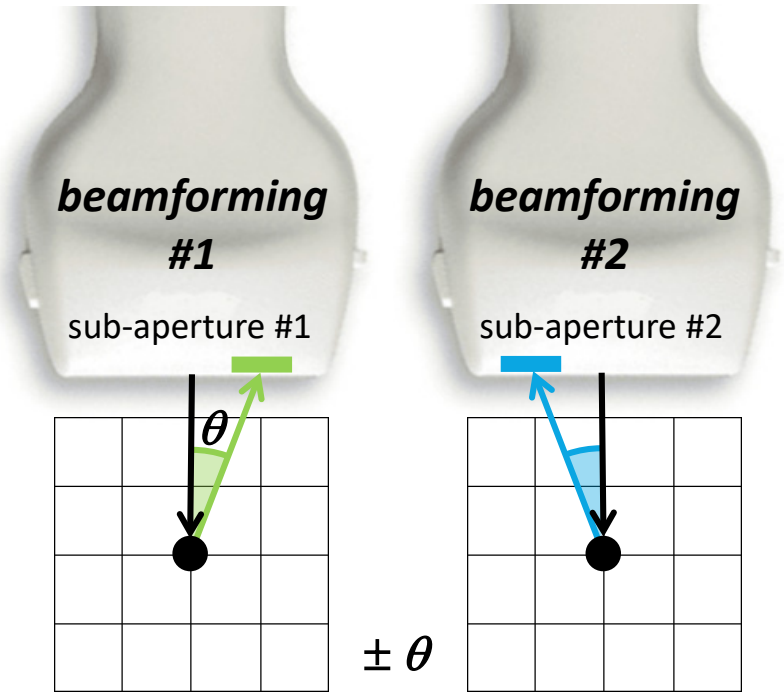
- Ricci *et al*. IEEE TUFFC 2014;61:314-324

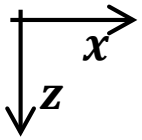
X-beam vector Doppler using plane waves

Use two sub-apertures during beamforming…



- unsteered signal transmissions
- 2 directional signal receptions





$$\begin{cases} v_z = \dfrac{v_{D1} + v_{D2}}{1 + \cos\theta} \\ v_x = \dfrac{v_{D1} - v_{D2}}{\sin\theta} \end{cases}$$

⚠ if angles are small… ⚠

Note:
the sub-aperture widths should be chosen so that the *f*-number ($= z/a$) is constant

key paper

- Madiena *et al*. IEEE TUFFC 2018;65:795-802

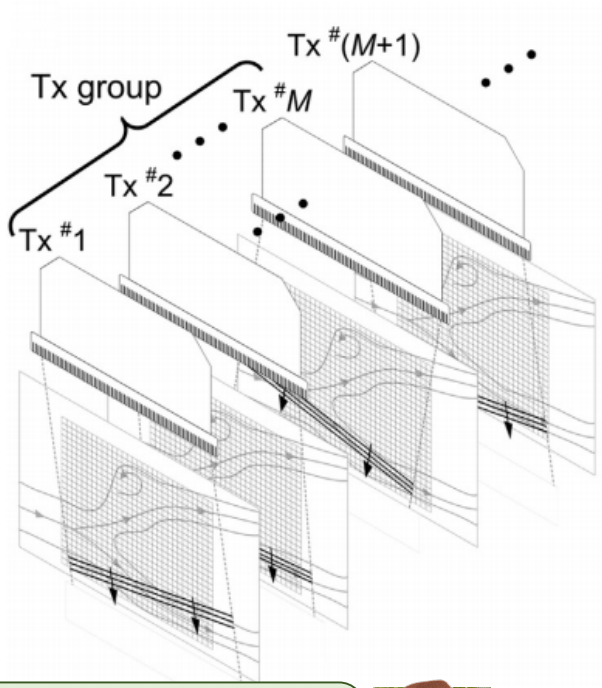
shooting-everywhere vector Doppler

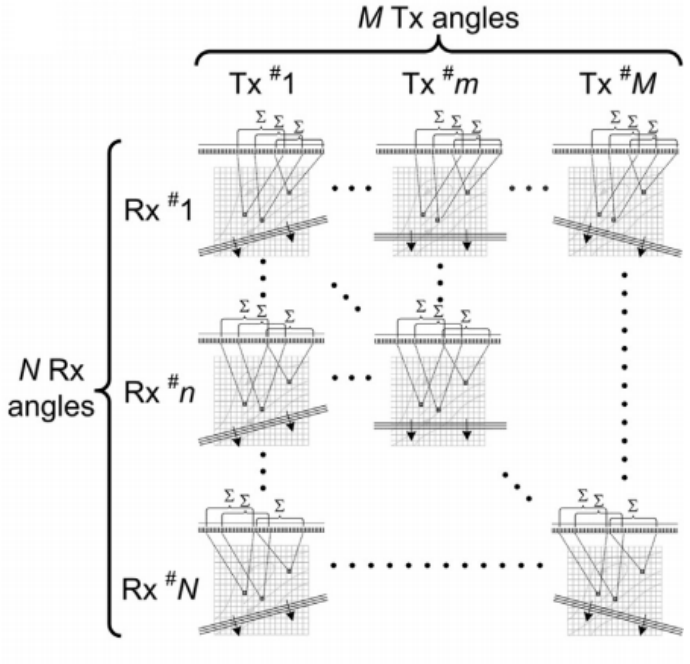
Use multiple transmits and multiple sub-apertures during beamforming…



- *M* steered signal transmissions
- *N* directional signal receptions







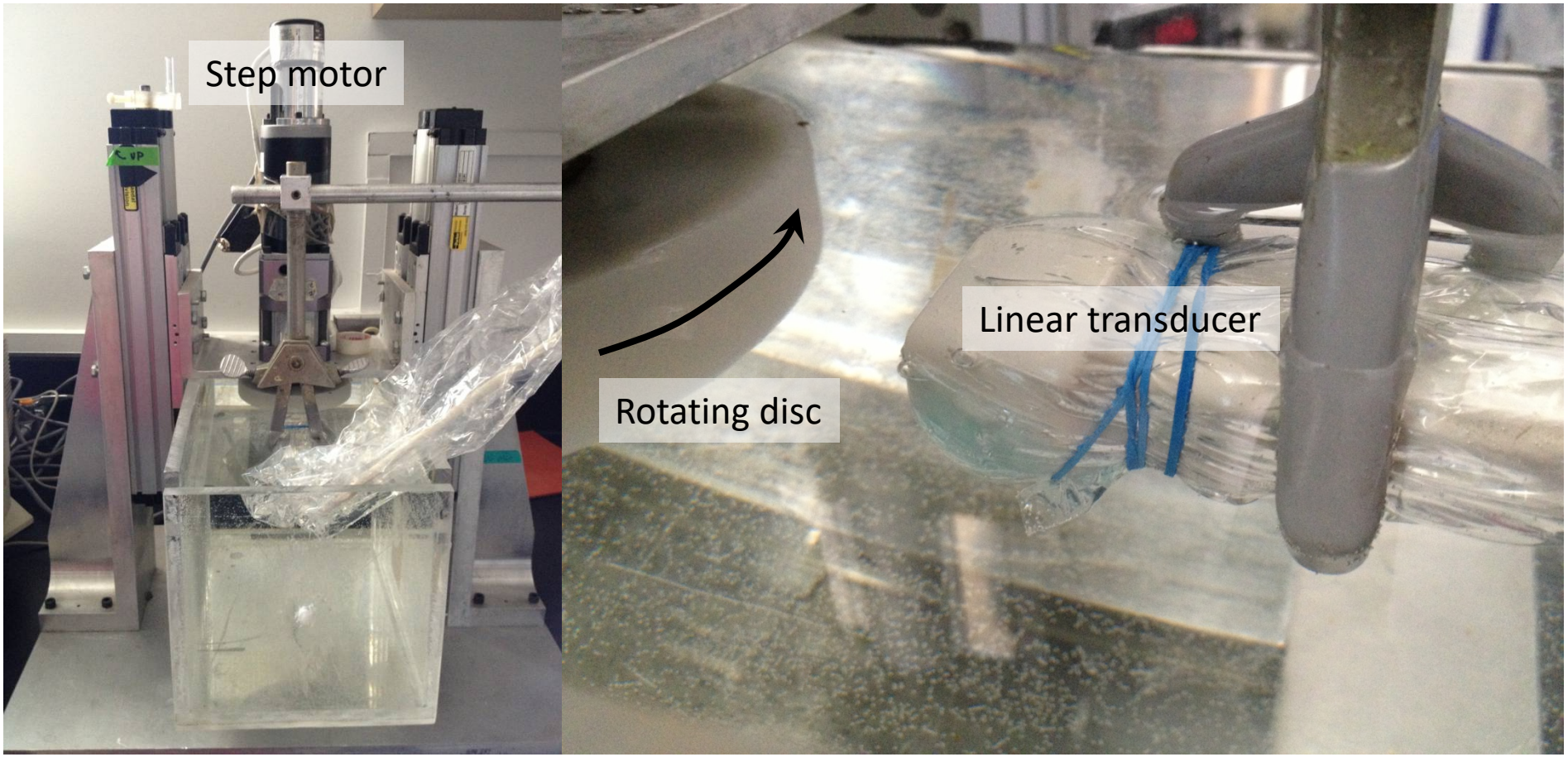
key paper

- Yiu and Yu. IEEE TUFFC 2016;11:1733-1744



Combine the *M*×*N* set into a linear system, and solve it in the least-squares sense

experiment: vector Doppler in a rotating disc

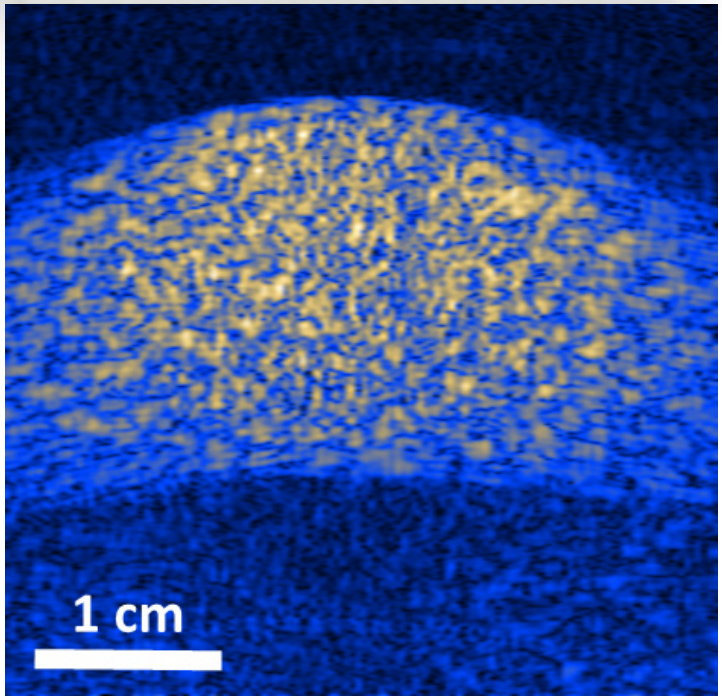


- 5-MHz linear array
- PRF = 10,000 Hz
- 32 transmits of non-steered plane waves

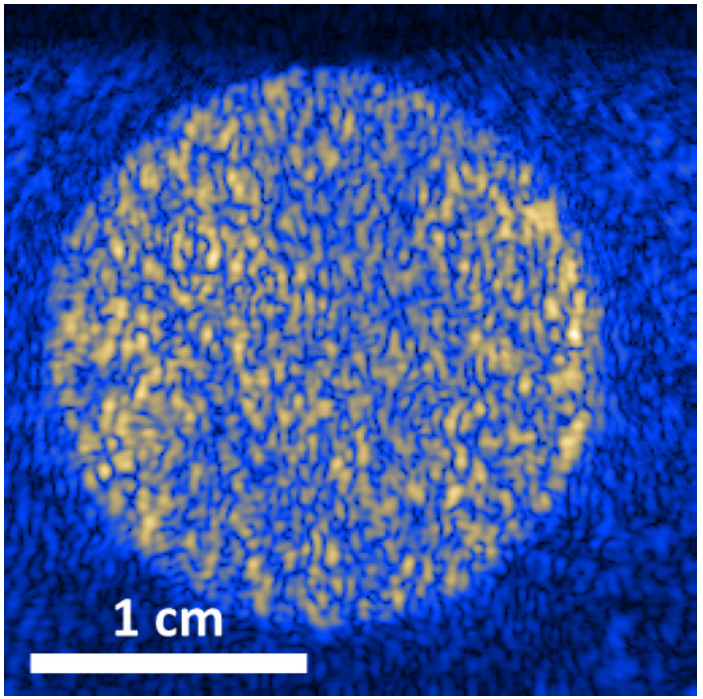
experiment: vector Doppler in a rotating disc (cont'd)

- **Dynamic focusing (using a delay-and-sum)**

Real envelopes of the raw I/Q

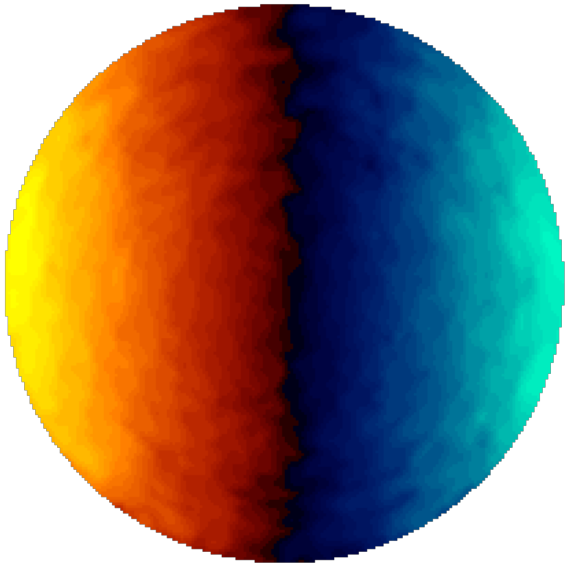


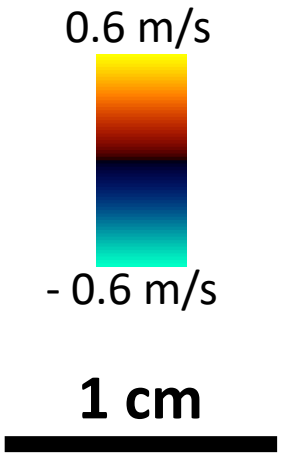
B-mode image



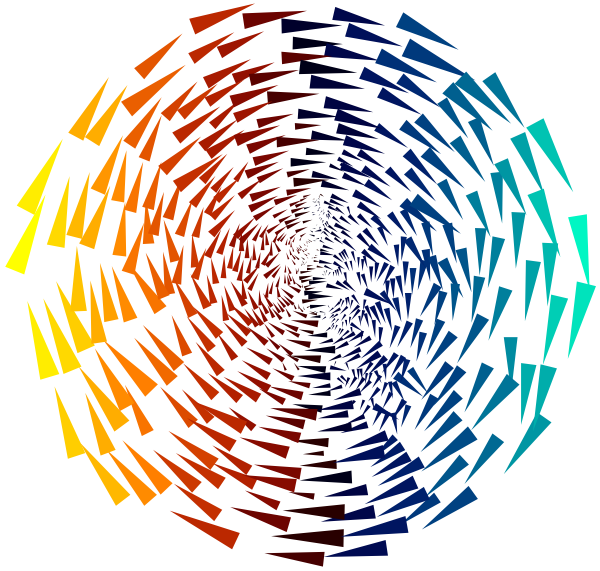
experiment: vector Doppler in a rotating disc (cont'd)

Color Doppler





vector Doppler



DIY: hands-on session on color Doppler and vector Doppler

clutter filtering & smoothing

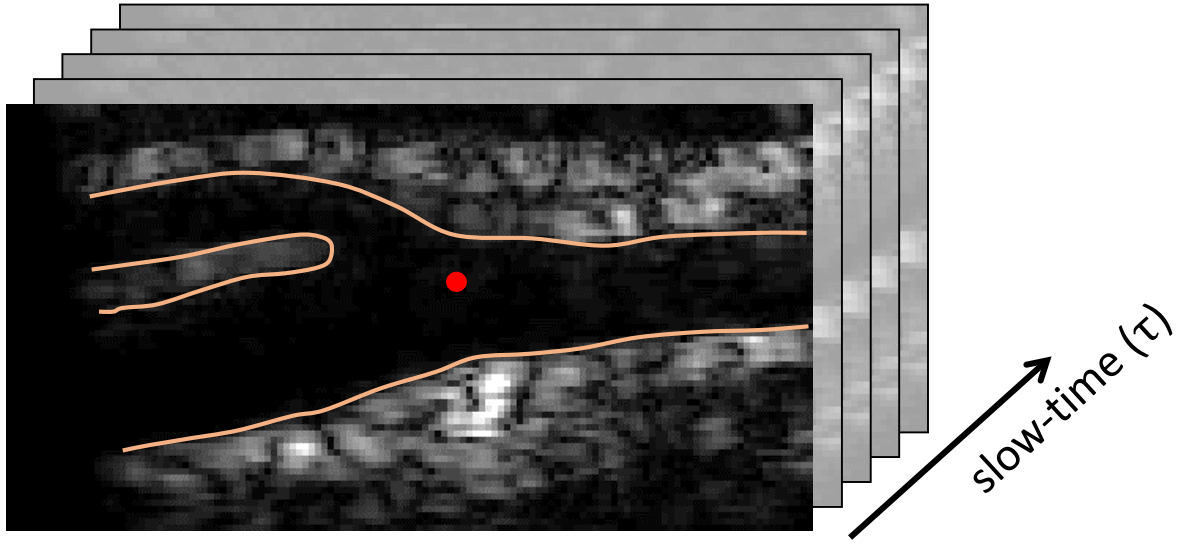
clutter filtering & smoothing

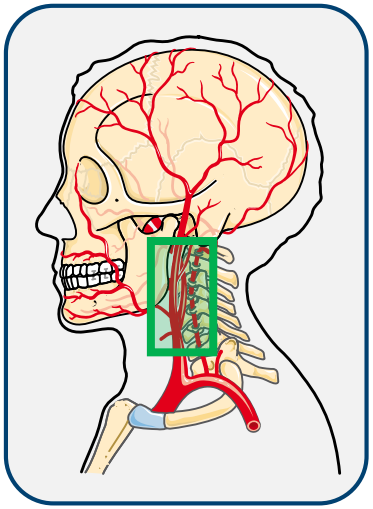
1. **Clutter filtering**: this pre-processing step is mandatory if you are interested in blood motion

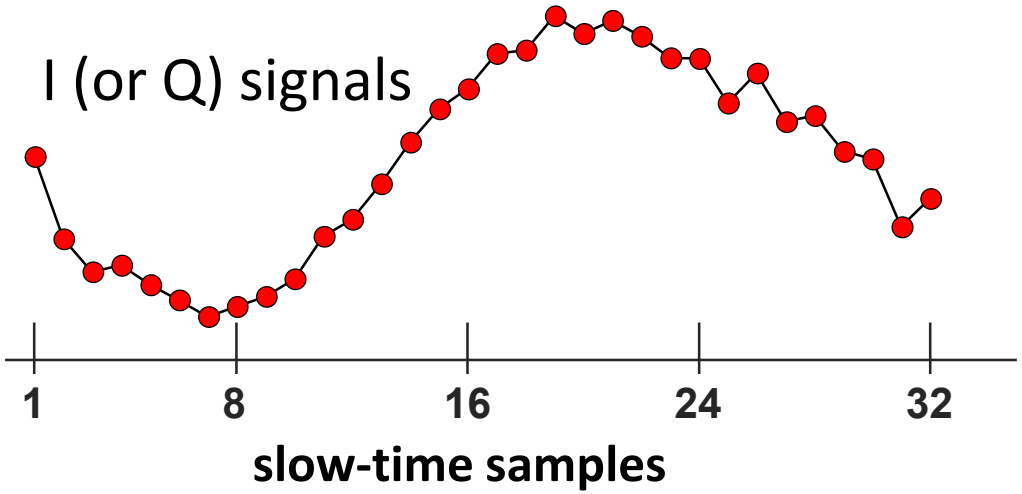
2. **Smoothing**: since data are spoiled by noise, clutter, approximations, algorithm simplifications, etc., this post-processing step is required if you are interested in analyzing your output data

⚠ These two steps are critical in blood flow imaging, regardless of the method used.
The results can be meaningless if these steps are not taken seriously.

I/Q signals over slow-time



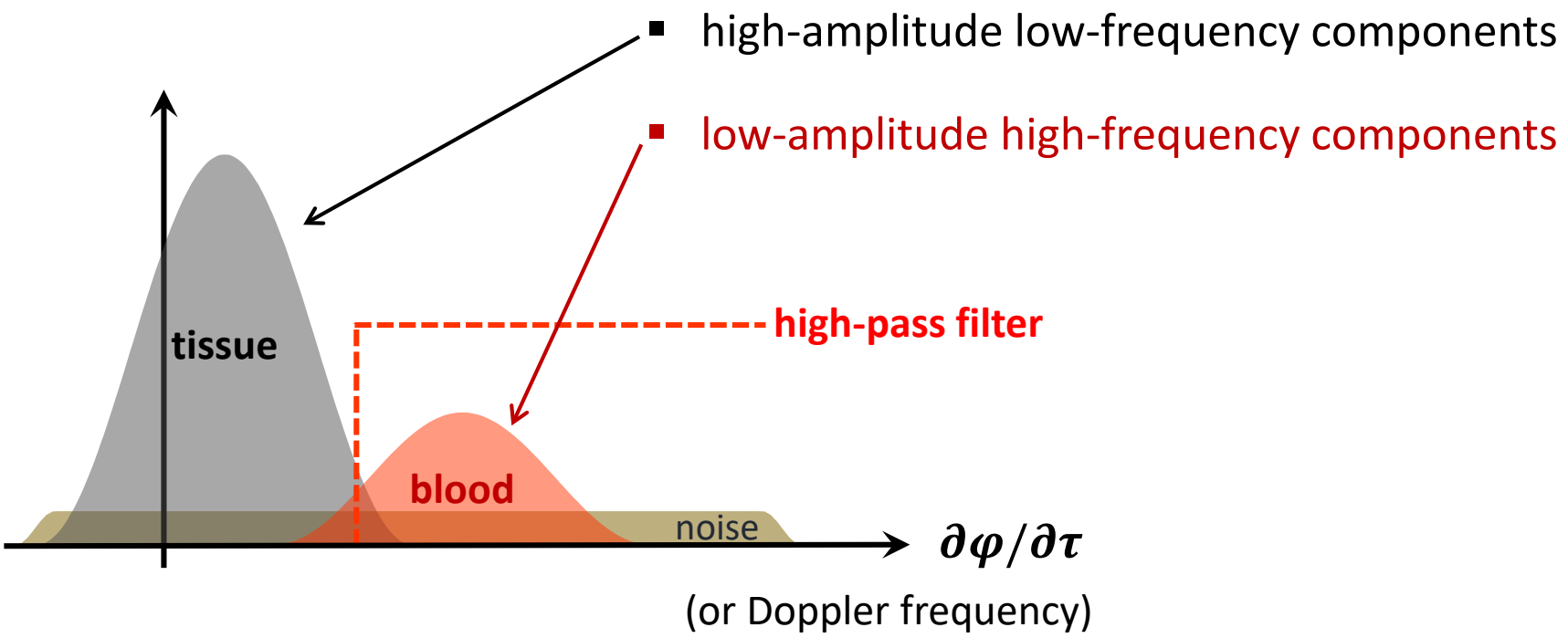




The “blood” signal is masked by a high-amplitude low-frequency signal (clutter)

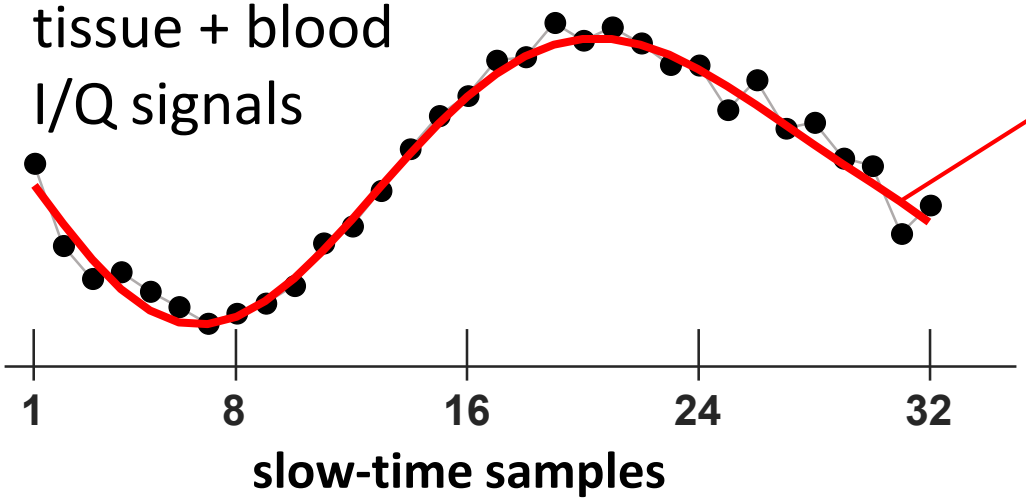
clutter (unwanted echoes)

- Blood signals are spoiled by clutter (unwanted echoes)
- Clutter is typically returned from surrounding tissues
- Clutter can cause serious performance issues in blood flow imaging



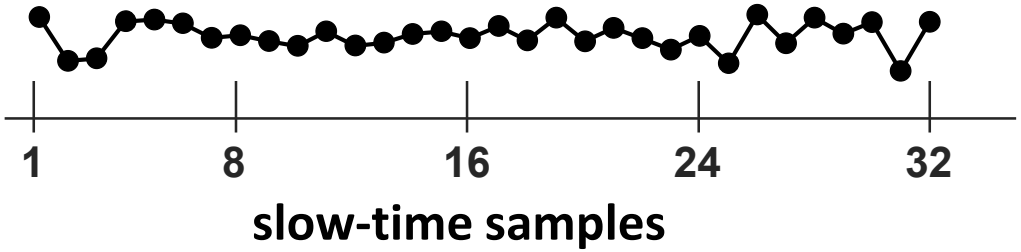
clutter filtering

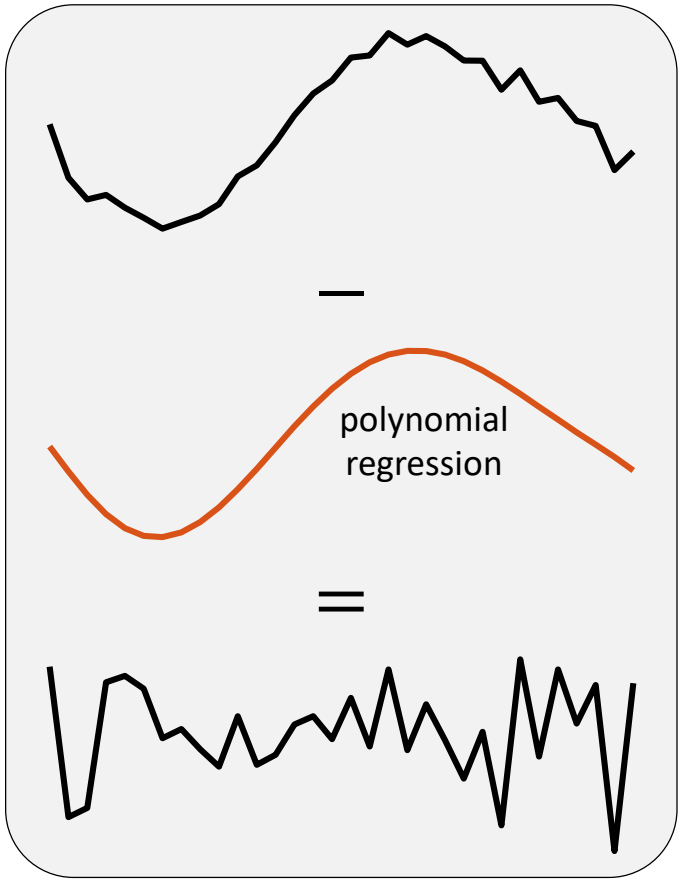
tissue + blood
I/Q signals



- Clutter signal, mostly issued from surrounding tissues
- Detected by low-pass filtering

blood
I/Q signals





polynomial regression clutter filter in Matlab

- **Polynomial regression clutter filter – a short Matlab code**



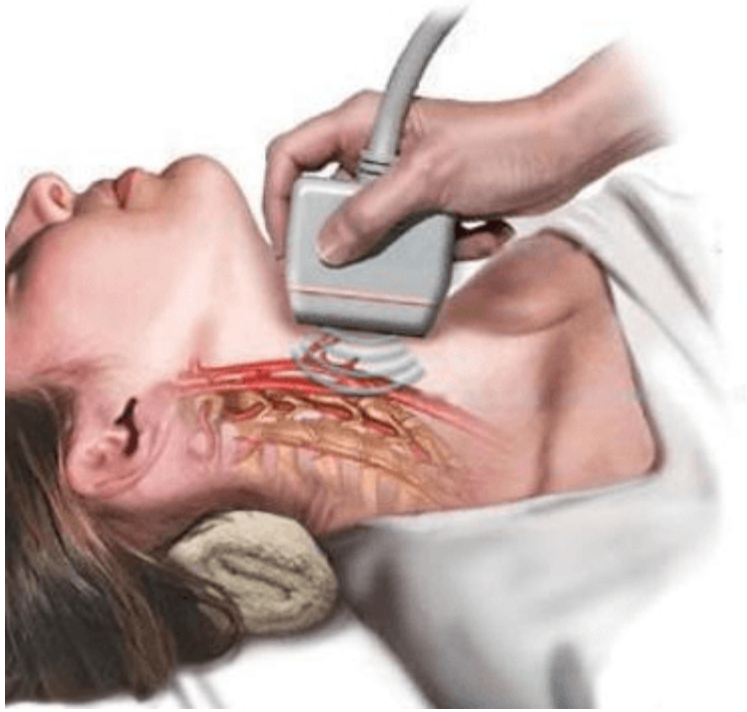
```
%-- IQ = I/Q ultrasound signals
% IQ is a 3-D array
% dim #1-2 = space; dim #3 = slow-time

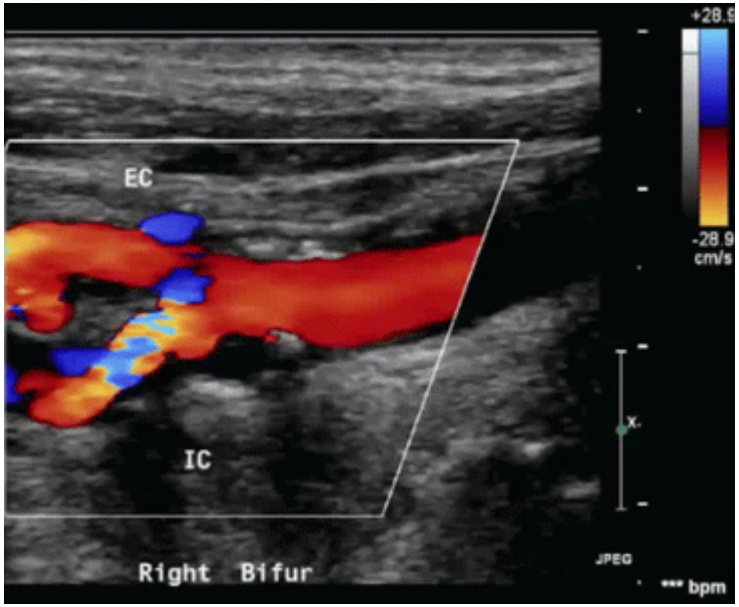
% DEG: degree of the polynomial

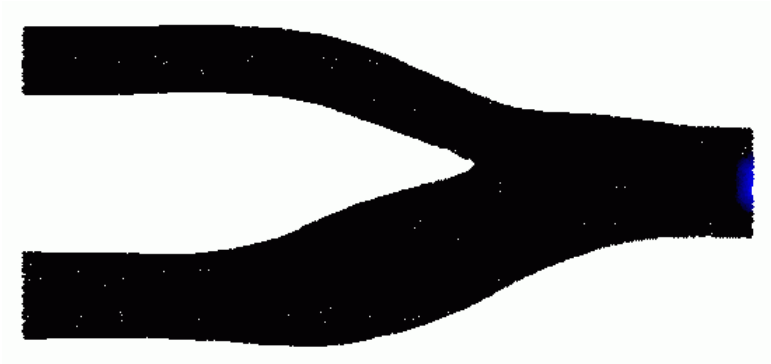
[m,n,q] = size(IQ);
t = reshape(1:q,[1 1 q])/q; % normalized slow-time

for k = 1:m*n
    i = mod(k-1,m)+1; j = (k-i)/m+1; % row & column #
    p = polyfit(t,IQ(i,j,:),DEG); % polynomial coeff
    IQ(i,j,:) = IQ(i,j,:)-polyval(p,t); % filtered I/Q
end
```

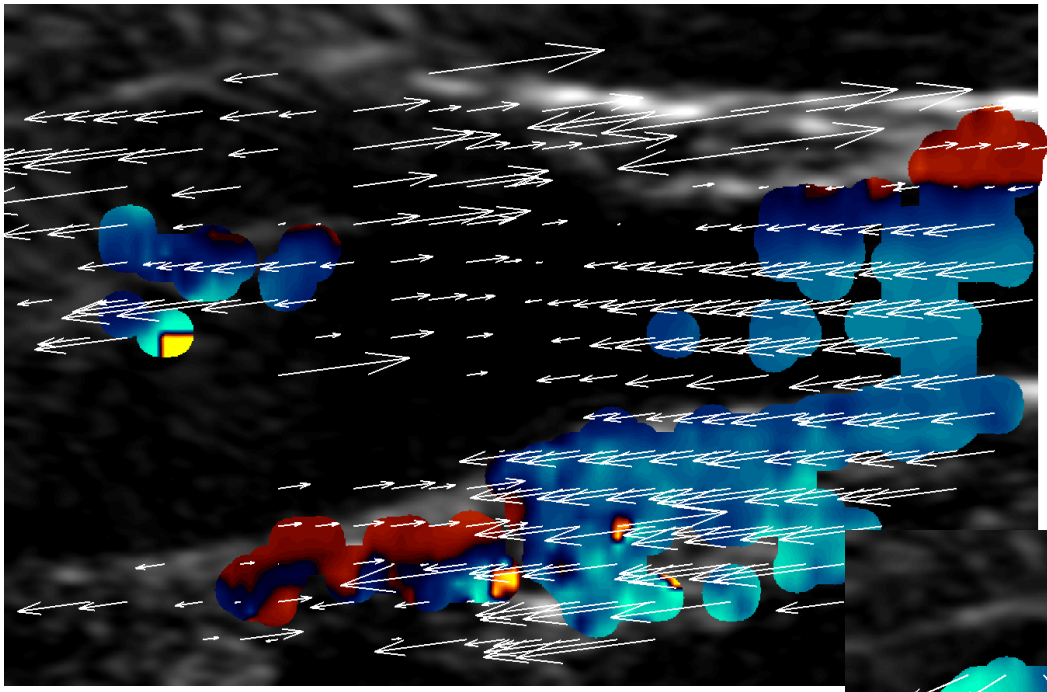
illustration: vector flow imaging in the carotid







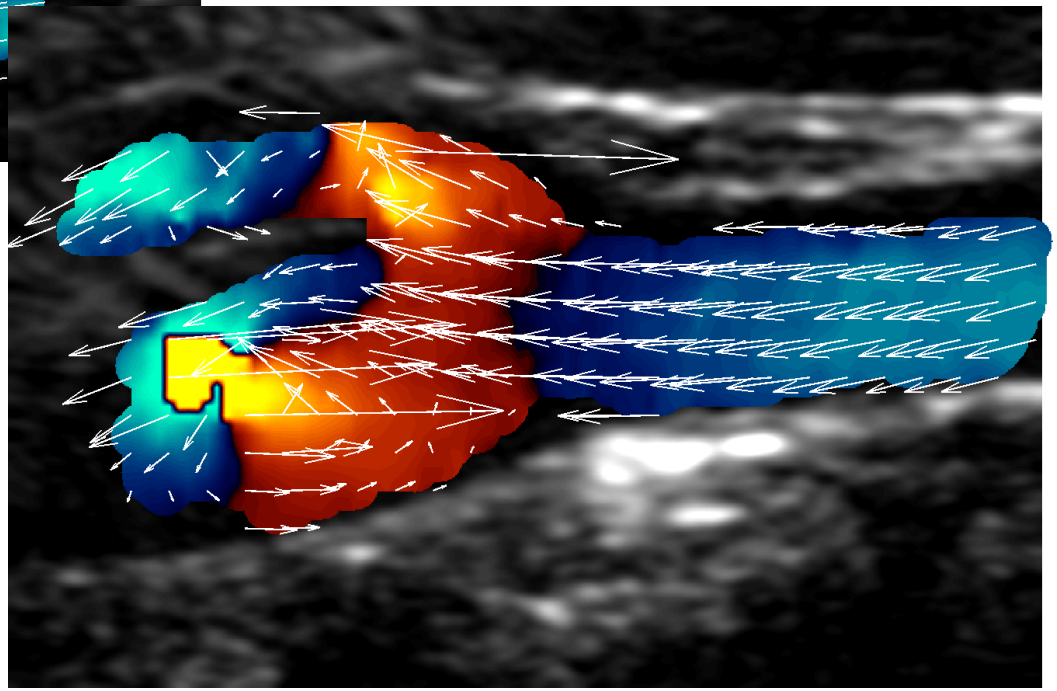
VFI in the carotid: clutter filtering



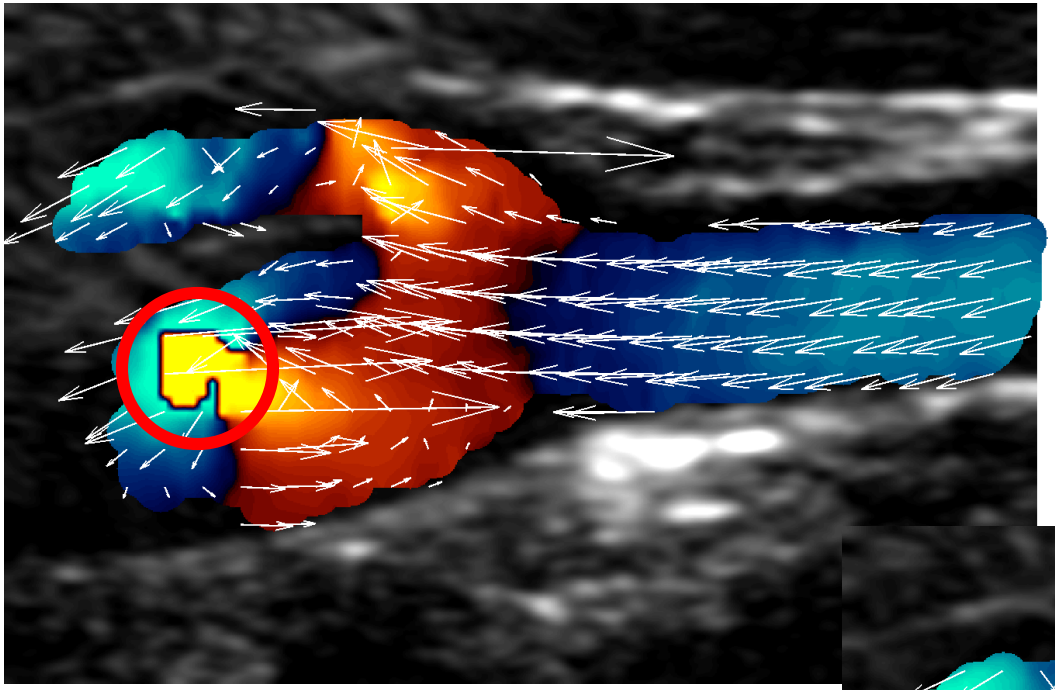
w/o clutter filtering

- adaptive polynomial regression

w/ clutter filtering



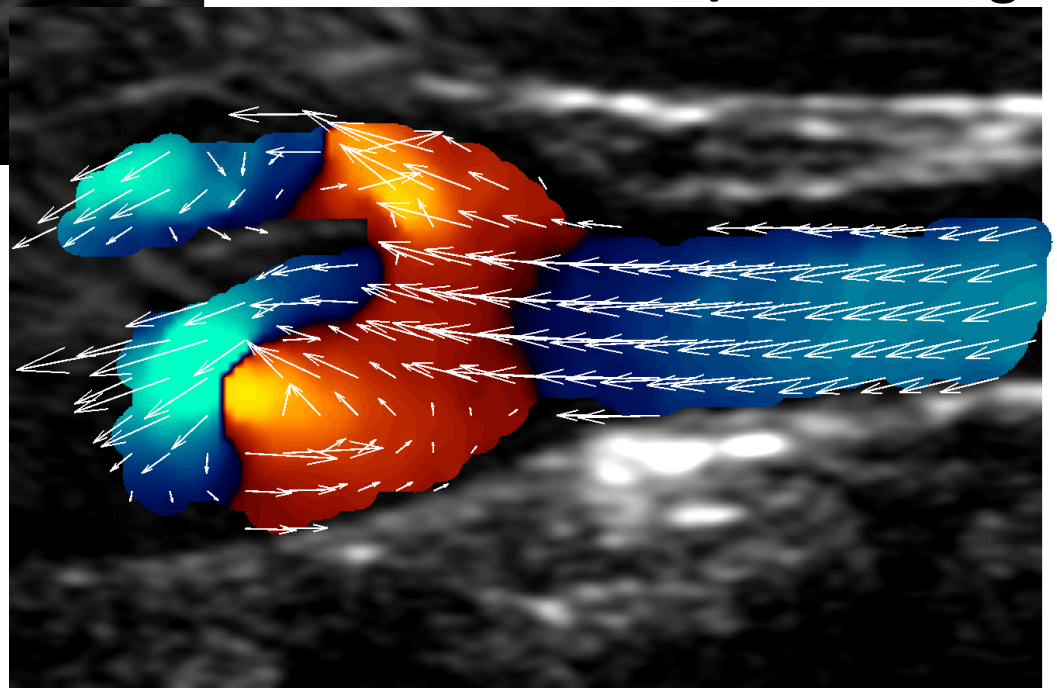
VFI in the carotid: dealiasing



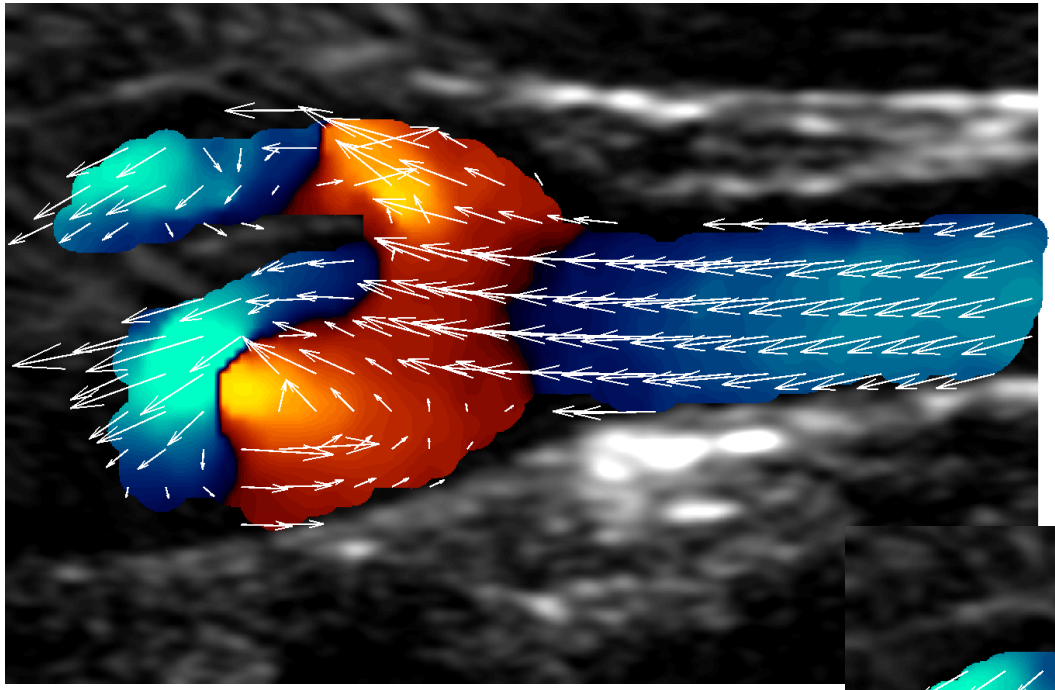
w/o dealiasing

- staggered dual-PRF

w/ dealiasing



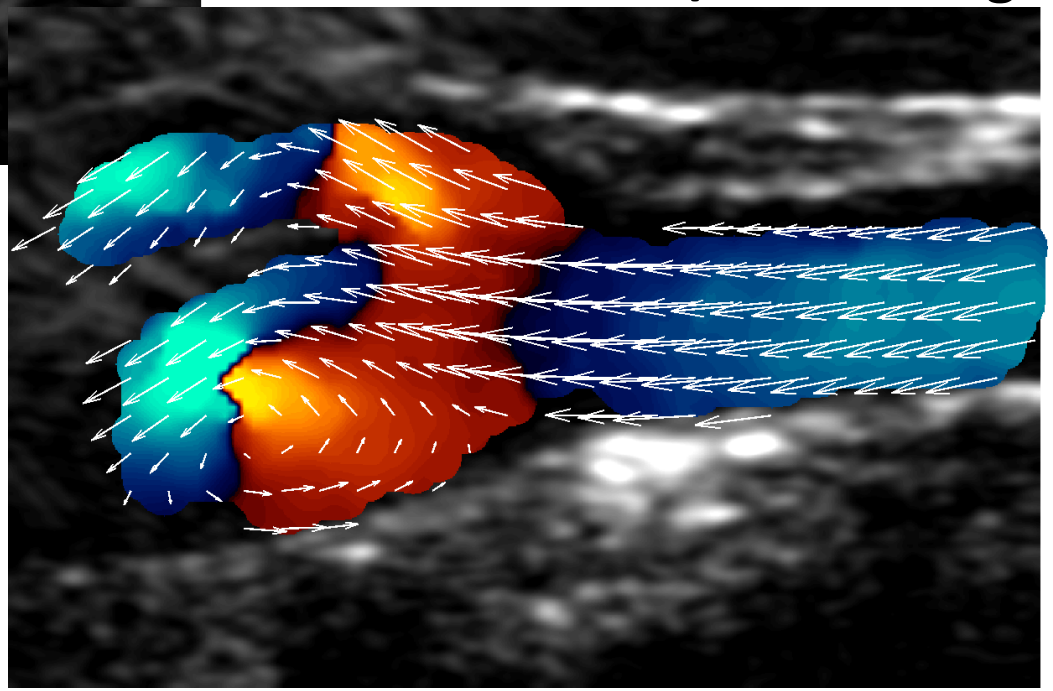
VFI in the carotid: smoothing



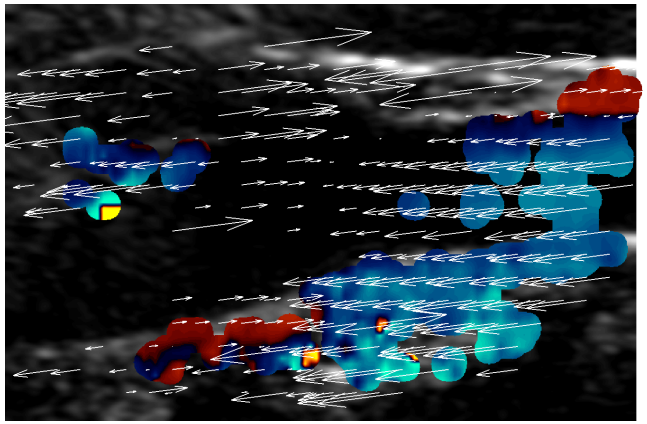
w/o smoothing

- automatic L_2-regularization

w/ smoothing

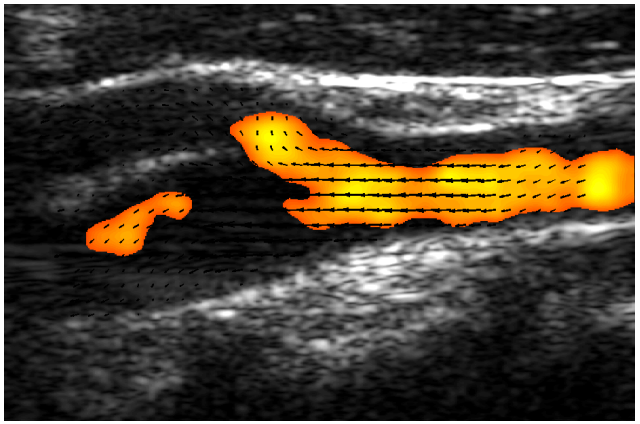


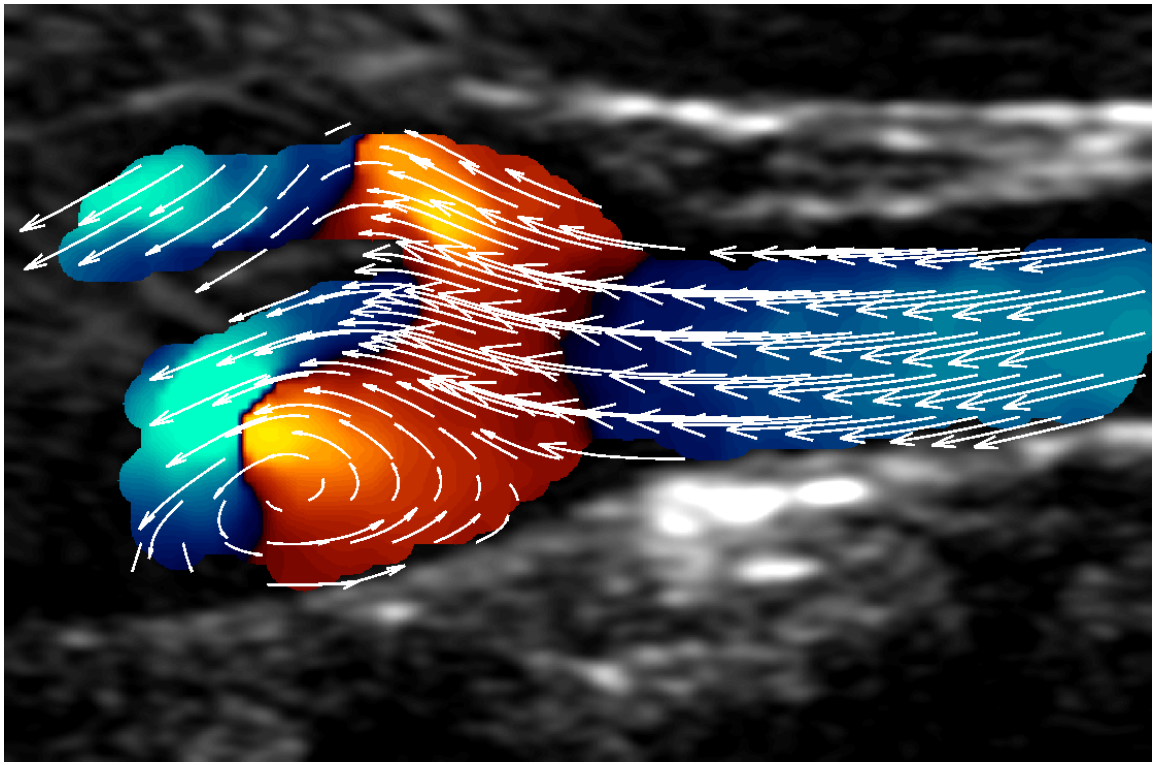
VFI in the carotid: recap



Three key processes:

1. **clutter filtering**
2. **dealiasing**
3. **smoothing**





clutter filtering & smoothing: some recommendations

There are many algorithms for clutter filtering and smoothing, from basic to advanced.

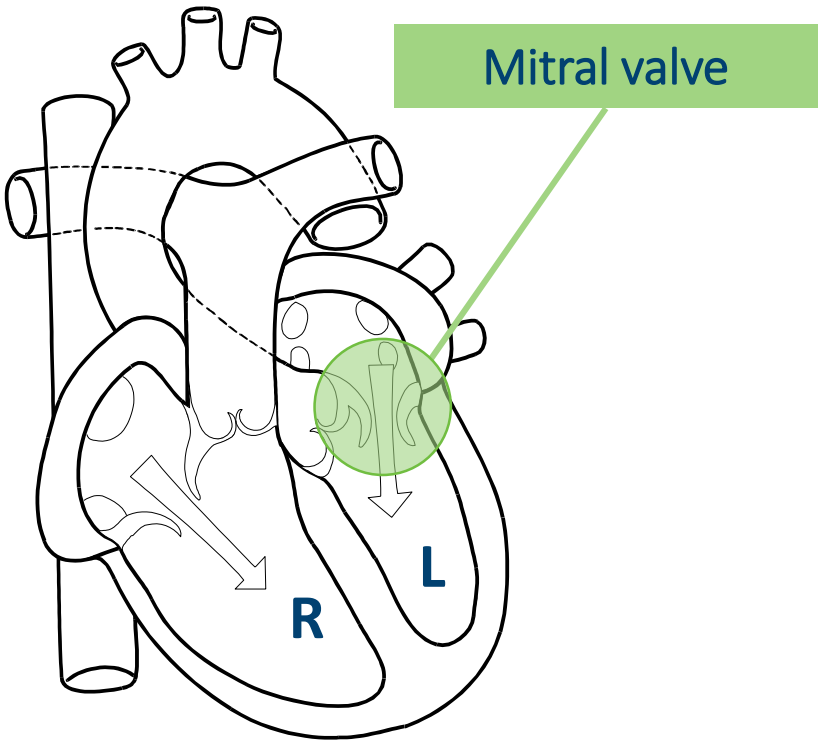
Recommendations

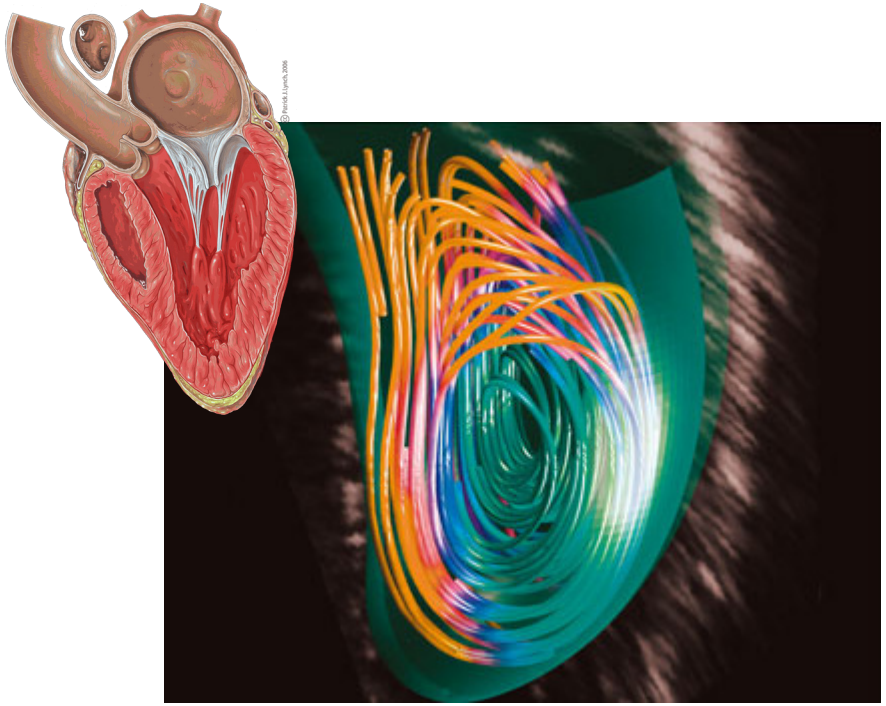
- PCA-based clutter filtering are powerful in most situations.
  drawback: not yet ready for real-time
- Avoid over-smoothing
- When thresholds or parameters must be tuned, use automatic methods to maximize inter-operator reproducibility

intraventricular vector flow mapping

diastolic vortex

- Diastolic filling is characterized by the formation of a large vortex
- The diastolic vortex rotates in the natural flow direction





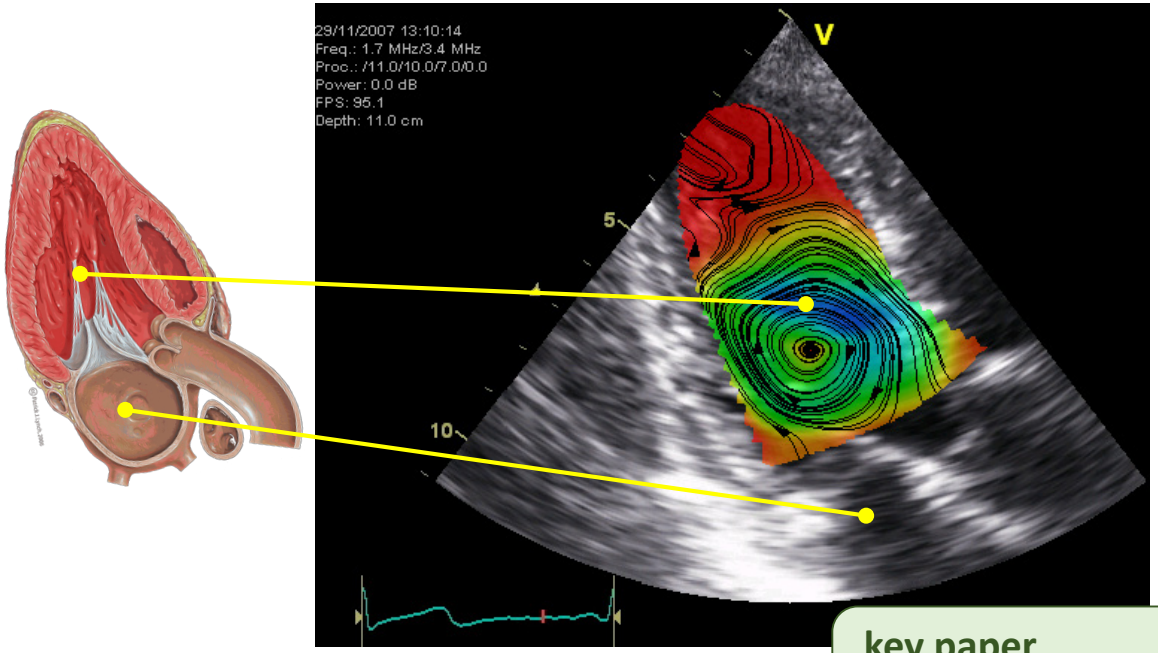
Pedrizzetti *et al.*, cover of *Nature Rev. Cardiol.*

diastolic vortex (cont'd)

- Ensures flow transition to ejection
- Minimizes fluid energy dissipation
- Optimizes left ventricular myocardial efficiency

from simulations

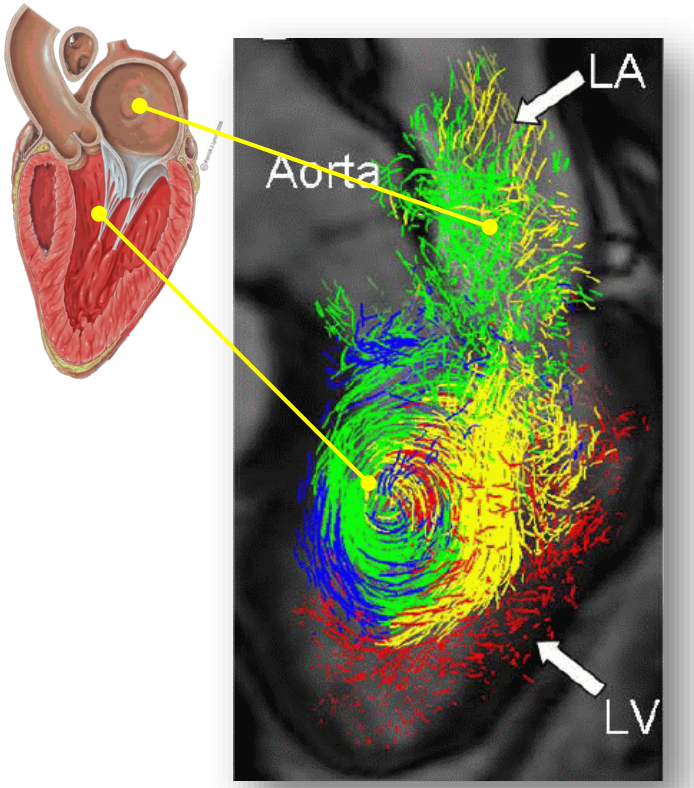
- **Vortex patterns are likely a key factor of diastolic function**



key paper

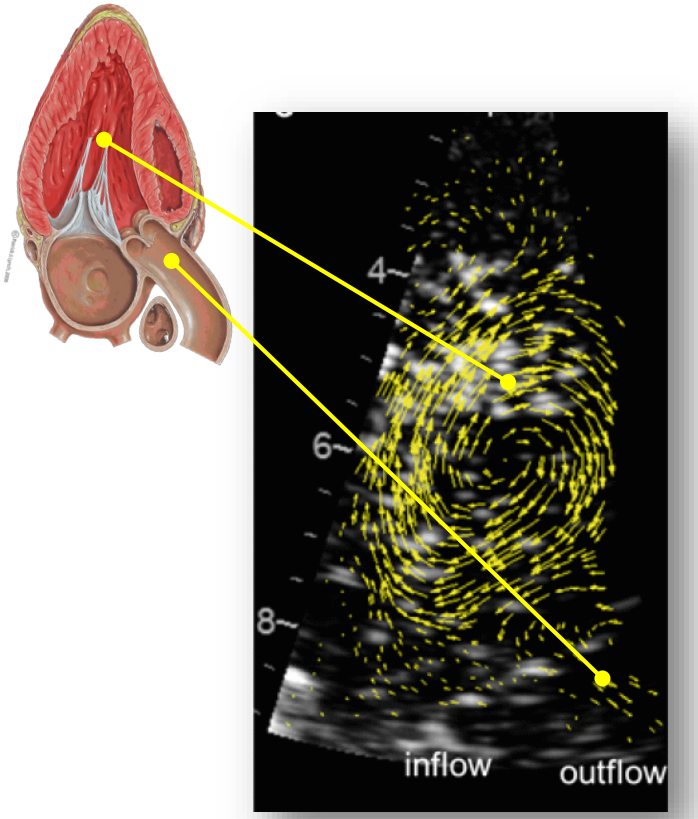
- Garcia *et al*. IEEE Trans Med Imag 2010;29:1701-1713

Cardiac Magnetic Resonance



Eriksson *et al*.
J. Cardiov. Magn. Reson. 2010

echo-PIV

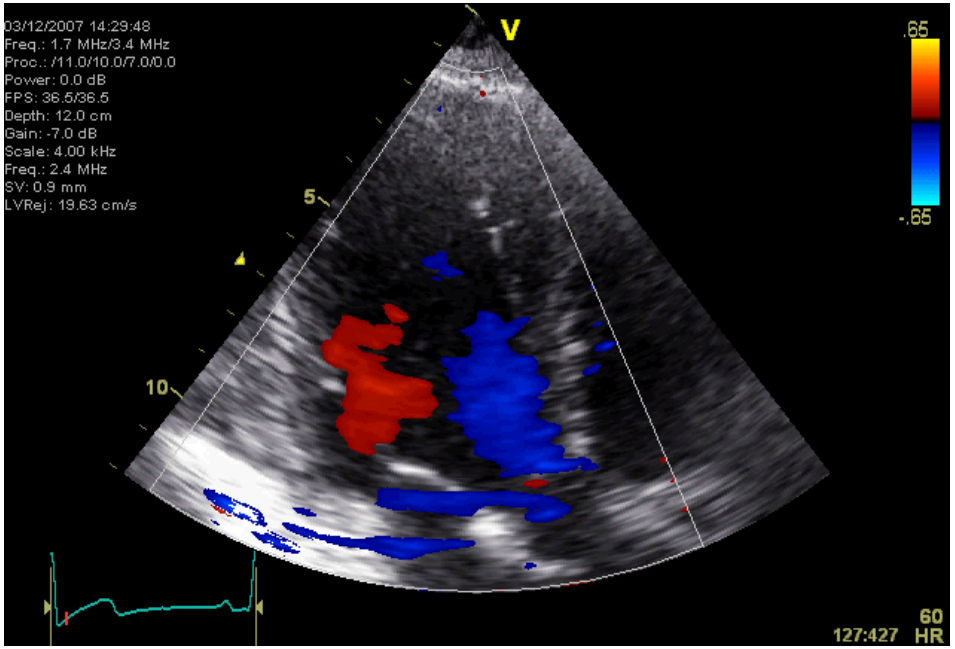


Hong *et al*.
JACC Cardiovasc. Imag. 2008

Problem: CMR and echo-PIV cannot be used in routine clinical practice

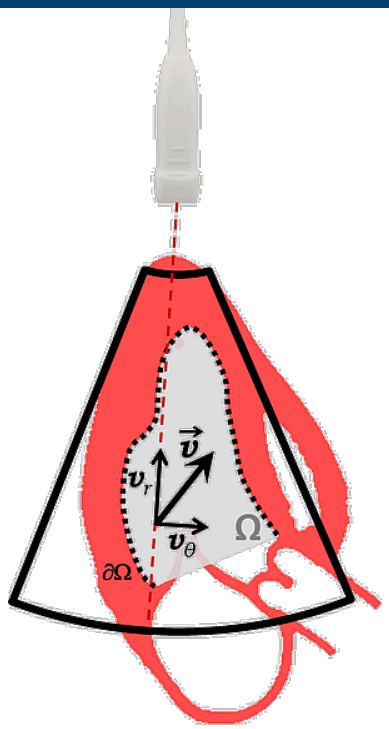
clinical Doppler echocardiography

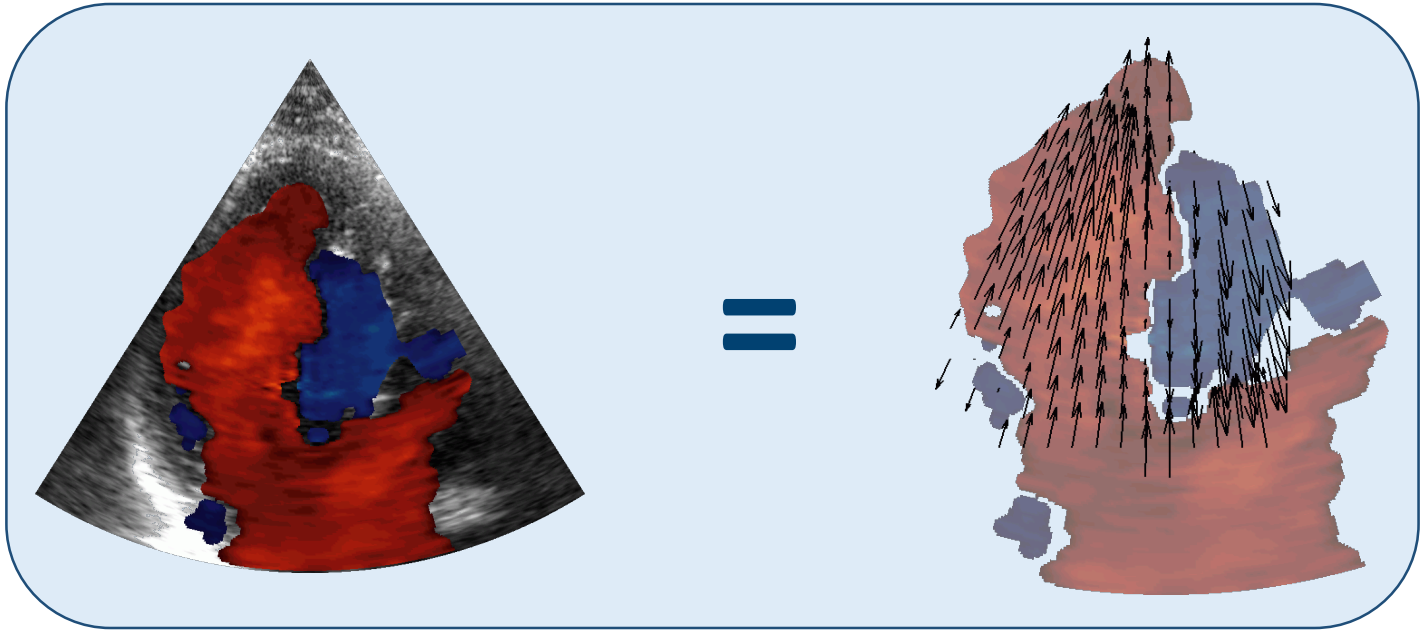






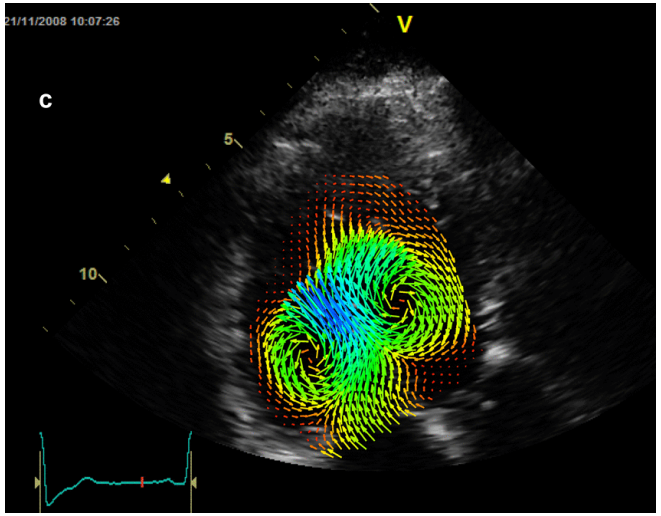
clinical Doppler echocardiography





Problem: clinical Doppler echocardiography returns a scalar field

"Vector Flow Mapping"



Garcia *et al.*
IEEE Trans Med Imag. 2010

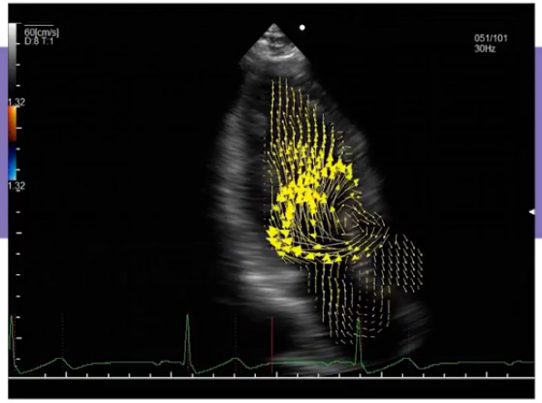


Hitachi Aloka Medical America, Inc.

HITACHI
Inspire the Next

VFM
Vector Flow Mapping

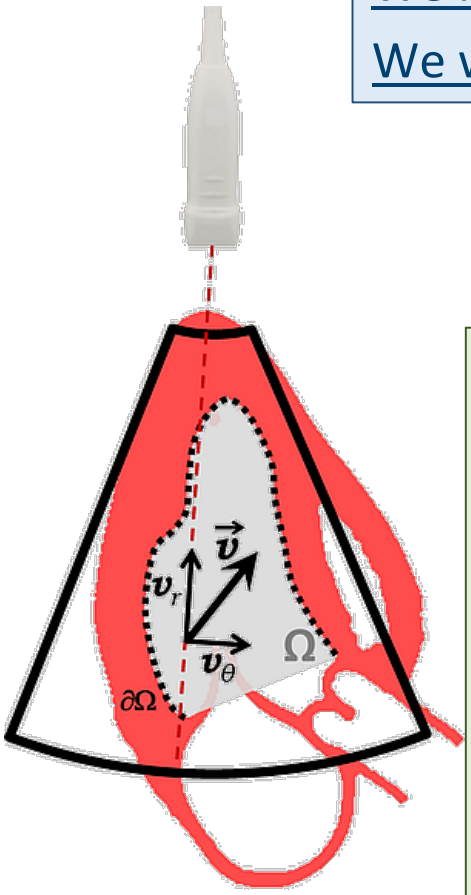
Evaluates Flow Dynamics in the Heart and Vessels in a Whole New Way



www.youtube.com/watch?v=XjIHQc3ASxA

We have: a **noisy scalar** field of Doppler velocities in a polar grid

We want: a **smooth** velocity **vector** field in this polar grid

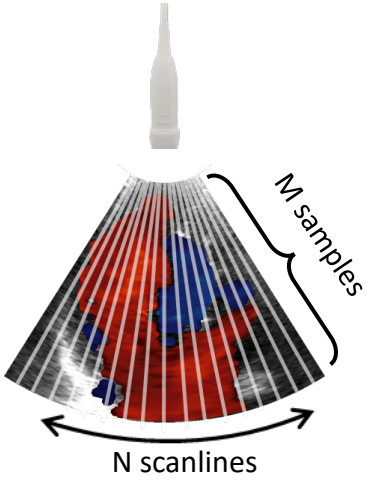


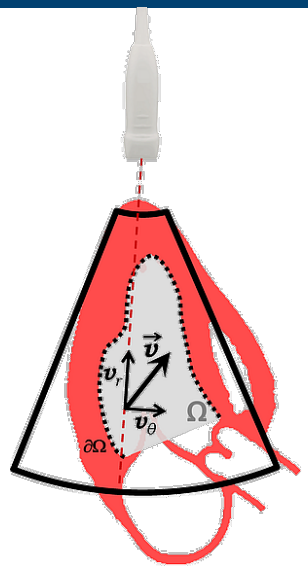
1. the radial velocities must be ≈ Doppler velocities

to make the problem well-posed, we add some constraints:

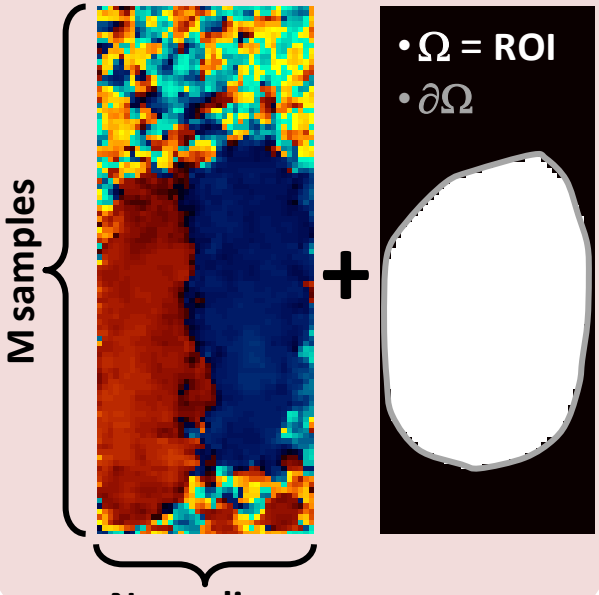
2. the quantity of mass is conserved
3. the boundary flow is parallel to the wall
4. the vector field is smooth

basics of *i*VFM

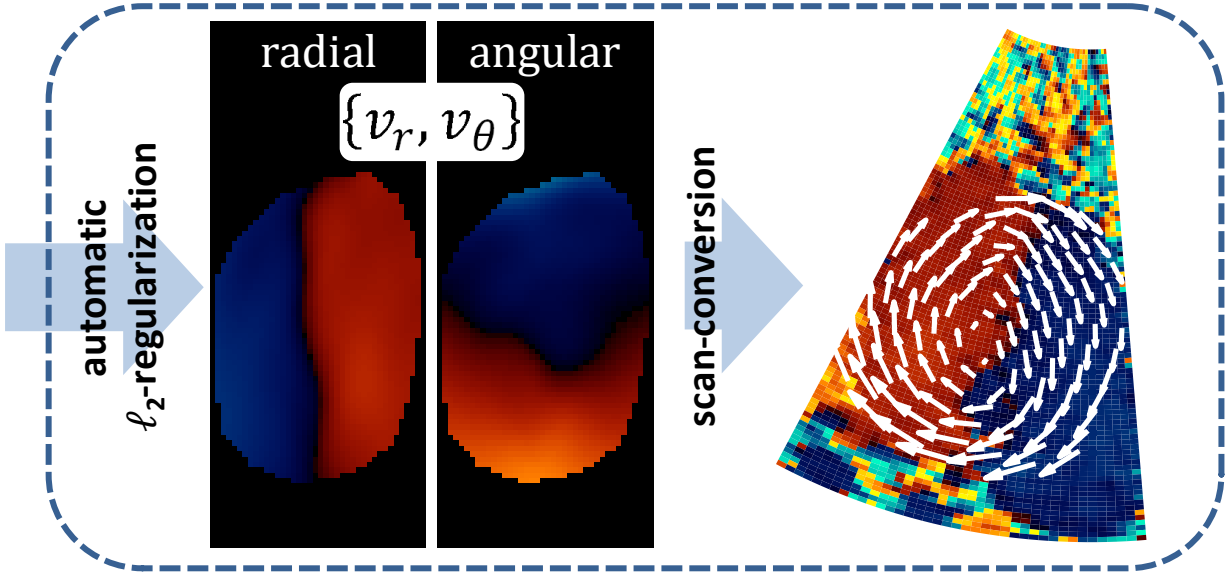




color Doppler echocardiography



*i*VFM – intraventricular Vector Flow Mapping



Assi *et al.*
Phys Med Biol. 2017

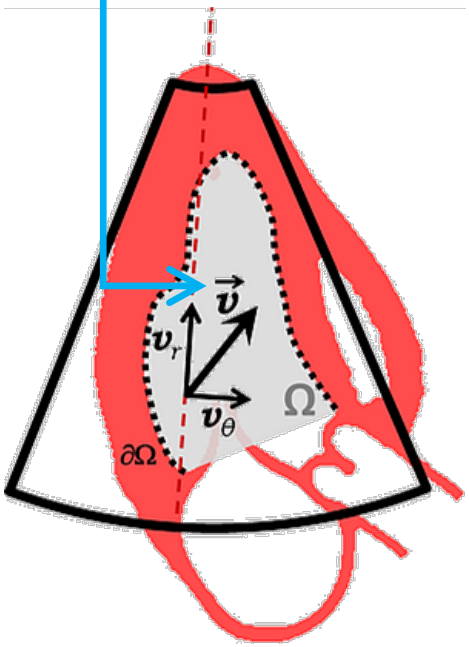
regularization problem

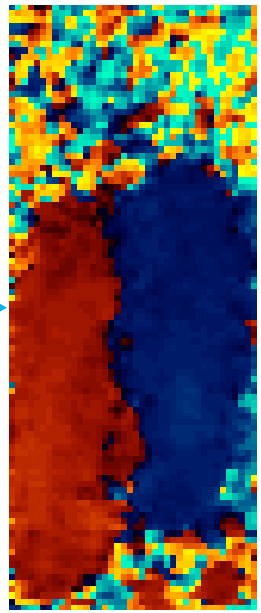
the velocity field to be recovered is the minimizer of this cost function:

$$J(\vec{v}) = J_0(\vec{v}) + \lambda_1 J_1(\vec{v}) + \lambda_2 J_2(\vec{v}) + \lambda_3 J_3(\vec{v})$$

$$= \underbrace{\int_\Omega (v_r - v_D)^2}_{\text{1) fit to the Doppler data}} + \lambda_1 \underbrace{\int_\Omega (r\partial_r v_r + v_r + \partial_\theta v_\theta)^2}_{\text{2) divergence-free constraint}} + \lambda_2 \underbrace{\int_{\partial\Omega} \left(\vec{v}\cdot\vec{d}_{wall}\right)^2}_{\text{3) boundary conditions}} + \cdots$$

$$\lambda_3 \sum_{m\in\{r,\theta\}} \underbrace{\int_\Omega (r^2\partial_r^2 v_m)^2 + 2\left(r\partial_{r\theta}^2 v_m\right)^2 + \left(\partial_\theta^2 v_m\right)^2}_{\text{4) smoothness constraint}}.$$





automatic selection of the λ_i:
***L*-hypercurve**

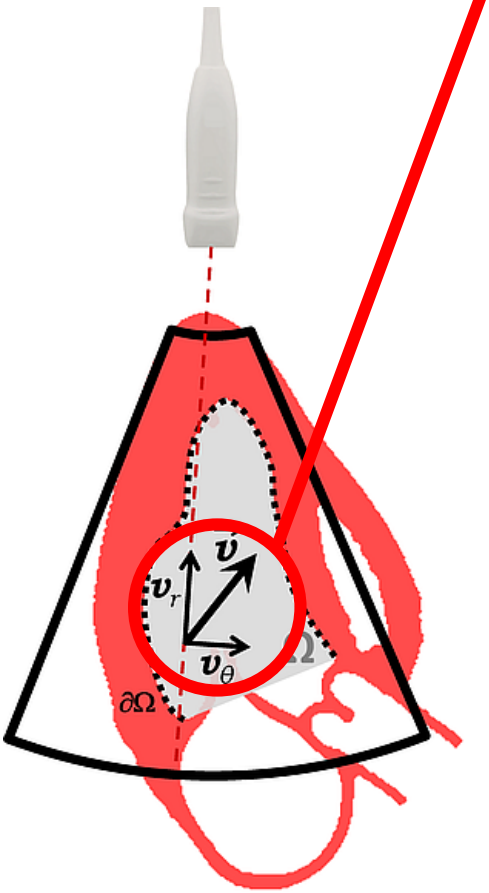
Belge *et al*. *Inverse Problems.* 2002

key paper

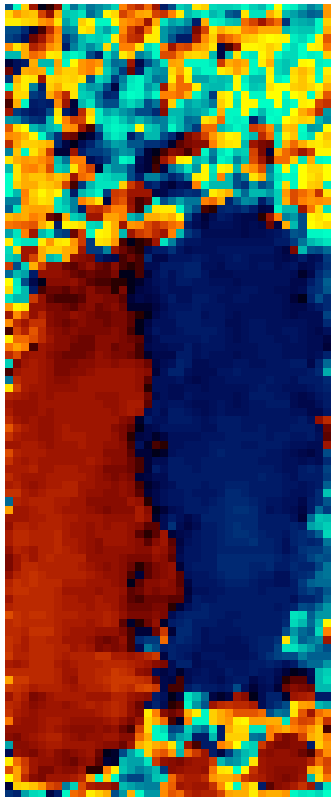
- Assi *et al*. Phys Med Biol 2017; 62:7131–7147

The discretized cost function can be written as:

$$J(\boldsymbol{v}) = (\boldsymbol{Q}_0\boldsymbol{v} - \boldsymbol{v}_\mathbf{D})^\mathrm{T}(\boldsymbol{Q}_0\boldsymbol{v} - \boldsymbol{v}_\mathbf{D}) + \sum_{l=1\ldots3} \lambda_l \, \boldsymbol{v}^\mathrm{T}\boldsymbol{Q}_l^\mathrm{T}\boldsymbol{Q}_l\boldsymbol{v}$$



Doppler raw data



$$\boldsymbol{Q}_0 = [1\ 0]\otimes\boldsymbol{I}_{MN}\,;$$

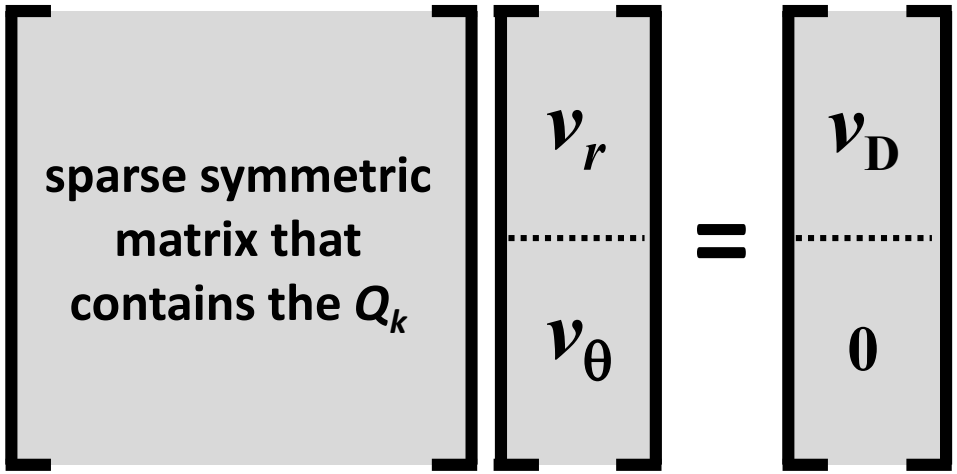
$$\boldsymbol{Q}_1 = \left[\frac{1}{h_r}\left(\boldsymbol{r}\,\mathbb{I}_{MN}{}^\mathrm{T}\right)\circ\left(\boldsymbol{I}_N\otimes\dot{\boldsymbol{D}}_M\right) + \boldsymbol{I}_{MN}\,,\quad \frac{1}{h_\theta}\dot{\boldsymbol{D}}_N\otimes\boldsymbol{I}_M\right];$$

$$\boldsymbol{Q}_2 = [\mathrm{diag}(\boldsymbol{n}_r),\quad \mathrm{diag}(\boldsymbol{n}_\theta)]\,;$$

$$\boldsymbol{Q}_3 = \begin{bmatrix} \boldsymbol{I}_2\otimes\frac{1}{h_r{}^2}\left(\left(\boldsymbol{r}^{\circ 2}\,\mathbb{I}_{MN}{}^\mathrm{T}\right)\circ\left(\boldsymbol{I}_N\otimes\ddot{\boldsymbol{D}}_M\right)\right) \\ \boldsymbol{I}_2\otimes\frac{2}{h_r h_\theta}\left(\left(\boldsymbol{r}\,\mathbb{I}_{MN}{}^\mathrm{T}\right)\circ\left(\dot{\boldsymbol{D}}_N\otimes\dot{\boldsymbol{D}}_M\right)\right) \\ \boldsymbol{I}_2\otimes\frac{1}{h_\theta{}^2}\left(\ddot{\boldsymbol{D}}_N\otimes\boldsymbol{I}_M\right) \end{bmatrix}.$$

It might look complicated…
But it reduces to a **sparse linear system** that can be written with a 10-line Matlab code.

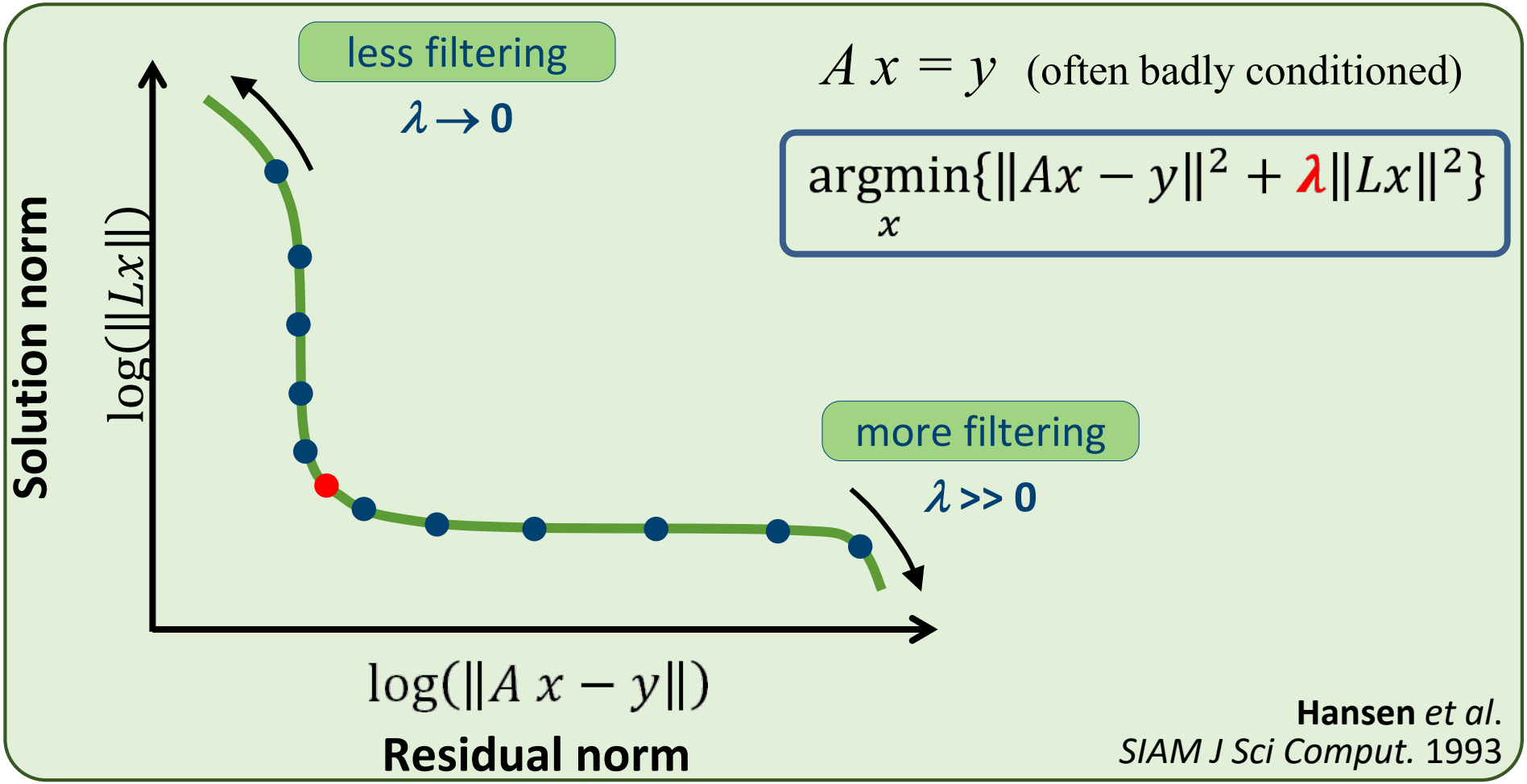
$$\boldsymbol{A}\boldsymbol{v} = \boldsymbol{b}, \text{ with}$$
$$\boldsymbol{A} = \boldsymbol{Q}_0^{\mathrm{T}}\boldsymbol{Q}_0 + \sum_{l=1\ldots3} \lambda_l \, \boldsymbol{Q}_l^{\mathrm{T}}\boldsymbol{Q}_l = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \boldsymbol{I}_{\boldsymbol{MN}} + \sum_{l=1\ldots3} \lambda_l \, \boldsymbol{Q}_l^{\mathrm{T}}\boldsymbol{Q}_l,$$
$$\text{and } \boldsymbol{b} = \boldsymbol{Q}_0^{\mathrm{T}}\boldsymbol{v}_{\mathrm{D}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \boldsymbol{v}_{\mathrm{D}}.$$



the *L*-hypercurve is a generalization of the *L*-curve

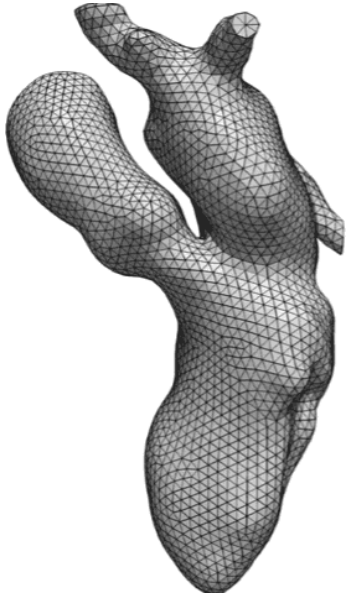
Belge *et al. Inverse Problems.* 2002

Objective: Find the best compromise oversmoothing *vs.* noise
i.e. find the best compromise bias / variance

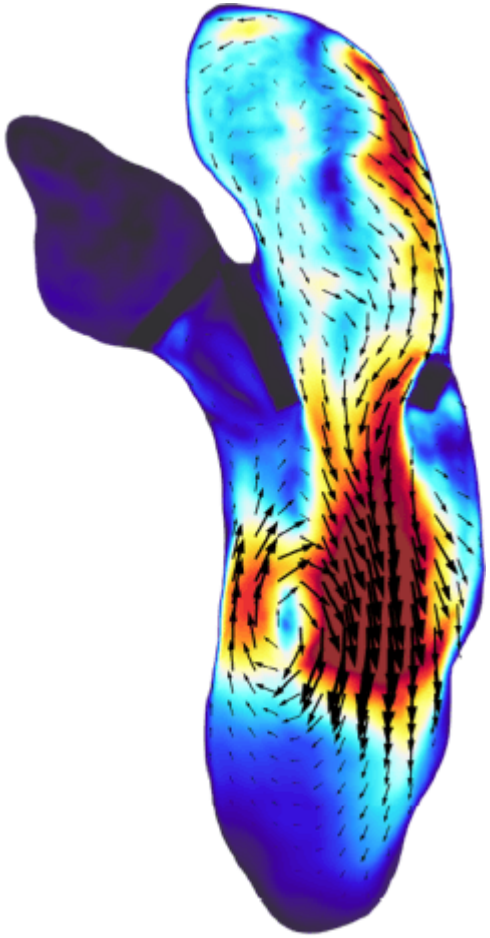


Hansen *et al.*
SIAM J Sci Comput. 1993

validation in a patient-specific CFD model



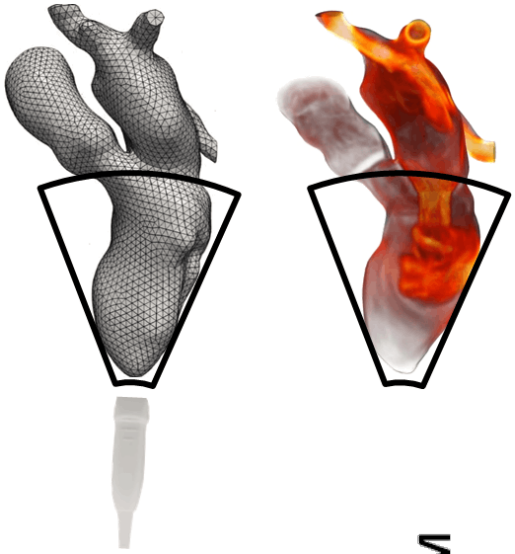


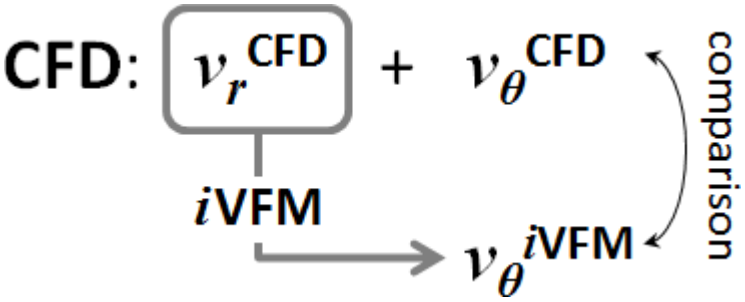


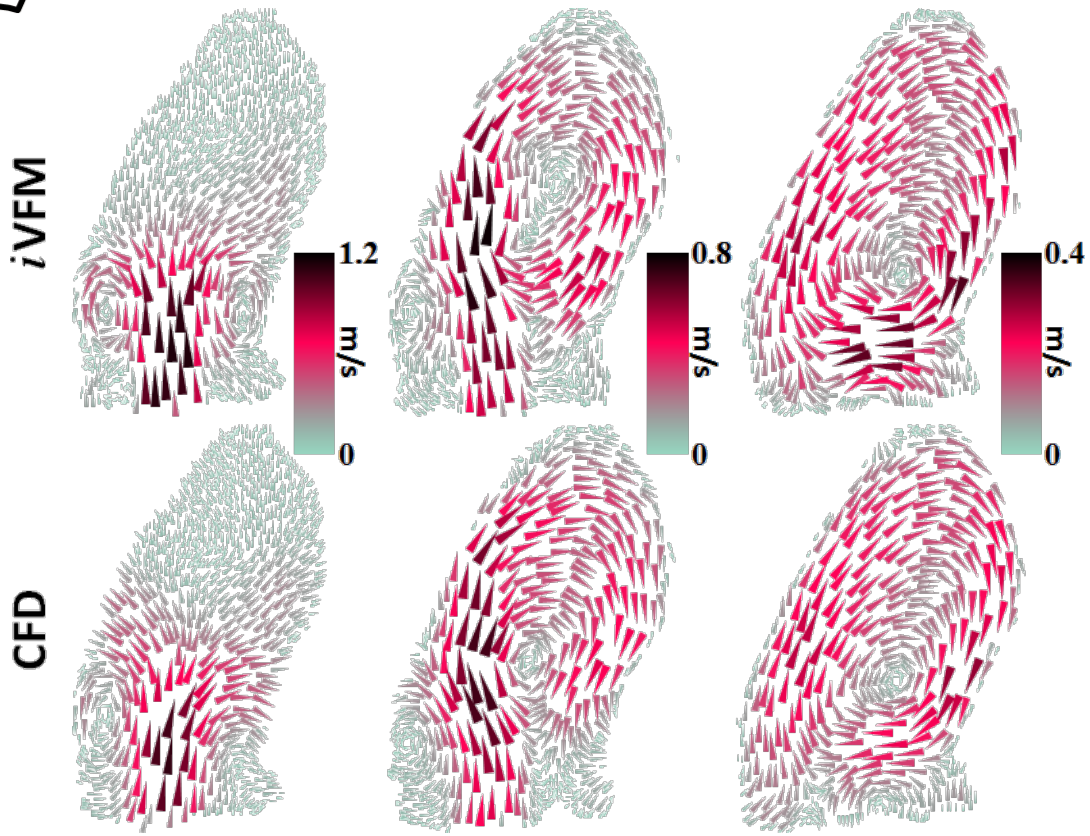
Chnafa *et al*.
Computers & Fluids. 2014
Ann Biomed Eng. 2016

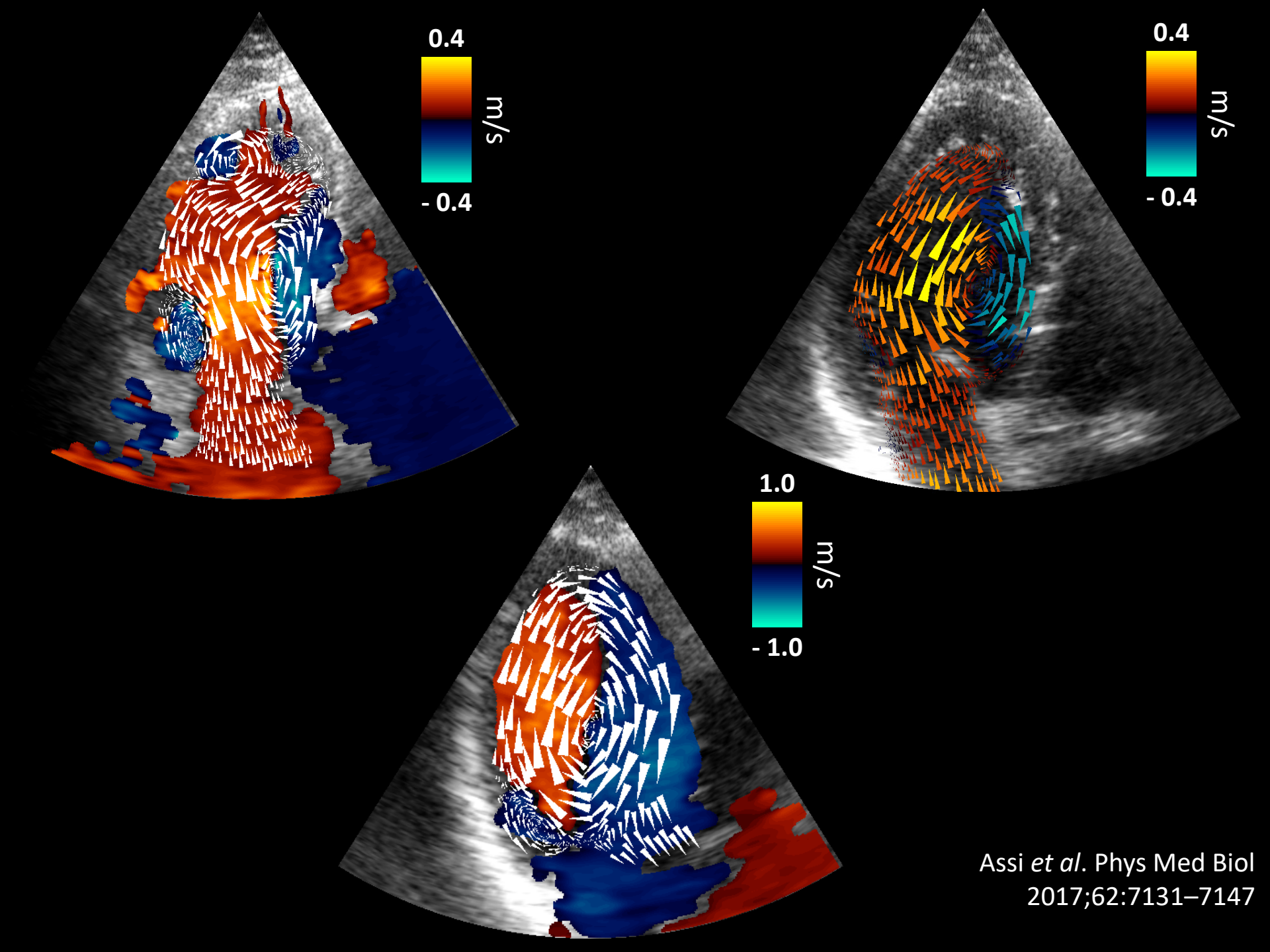
https://www.youtube.com/watch?v=DeOgt4kP2Ys

in silico results



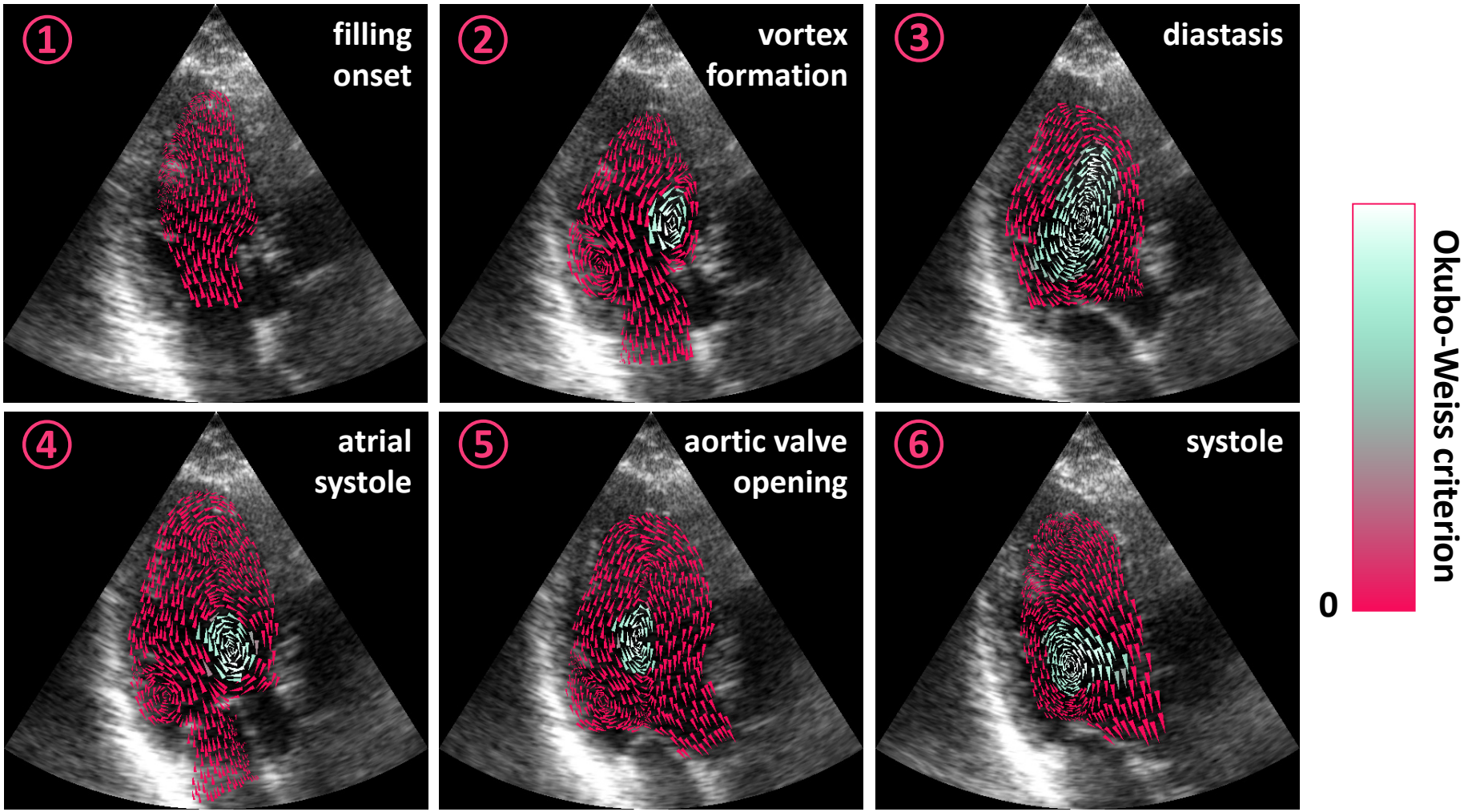






Assi *et al*. Phys Med Biol 2017;62:7131–7147

a clinical example



Assi *et al*. Phys Med Biol 2017;62:7131–7147